Homework 4 Solution – Chapter 5, problems 21 and 27.

21) The time to service customers at a bank teller's cage is exponentially distributed with a mean of 50 seconds.

(a) What is the probability that two customers in front of an arriving customer will each take less than 60 seconds to complete their transactions?
(b) What is the probability that two customers in front of will finish their transactions so that an arriving customer can reach the teller's cage within 2 minutes?

Service time is exponentially distributed with $\lambda=1/50$. You can either calculate the integral of the probability density function or use the cumulative distribution function.

$$F(x)=1-e^{-x/50}, x>0$$

(a) The probability that one customer I served within 1 minute is F(60)=.6988. Since customer service times are independent, the probability that two customers are each individually served within 1 minute is $(.6988)^2 = .4883$.
(b) The total service time of two customers could be calculated as the sum of exponential distributions. However, assuming that the service times are independent, this is also expressed as an Erlang distribution, which has a cumulative distribution

$$F(x)=1-\sum_{i=0}^{1}\left[e^{-x/50}\frac{(x/50)^i}{i!}\right], x>0$$

Calculating the result of this cumulative distribution, F(120)=.692

27) Suppose that cars arrive at a toll booth following the Poisson process with a mean arrival time of 15 seconds. What is the probability that up to one minute will elapse until three cars have arrived?

One could either treat the arrival process as Poisson or realize that interarrival times are exponential and the sum or exponential processes is Erlang. I'll give the Erlang solution.

Let X be the time until a car arrives. Using the Erlang cumulative distribution with $K\theta=4$ and x=1, the probability is

$$F(1)=1-\sum_{i=0}^{2}\left[e^{-4*1}\frac{(4*1)^i}{i!}\right], x>0$$

F(1)=.762