

**EE/PEP 345**

**Modeling and Simulation**

**Spring 2004**

**Class 2**

# Today's topics

- Simulation examples using a simulation table
  - Queuing system
    - Single server queue
    - Multi-server queues
  - Class project introduction

# System simulation using a simulation table

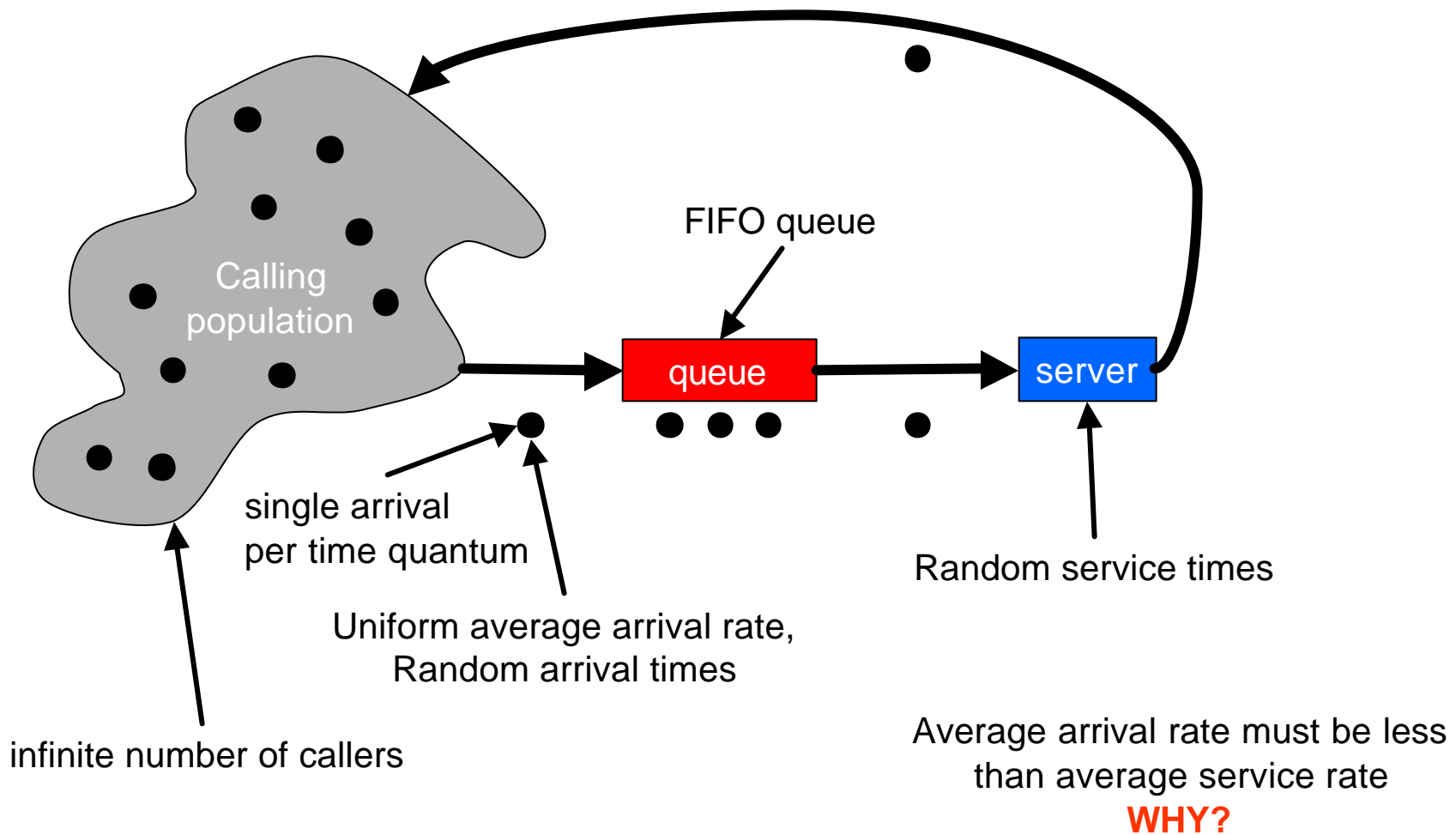
- Determine the characteristics of each input to the simulation, often inputs may be modeled as probability distributions
- Construct a simulation table: columns are inputs and response, rows are repetitions of system operation
- For each repetition, generate a value for each input, evaluate response

# Representative simulation table

	inputs							response
Repetition	$x_{i1}$	$x_{i2}$	$x_{i3}$	...	$x_{ij}$	...	$x_{ip}$	$y_i$
1								
2								
3								
4								
•								
•								
•								
n								

$x_{ik}$  is the  $k^{\text{th}}$  of  $p$  inputs for repetition  $i$   
 $y_i$  is the response for repetition  $i$

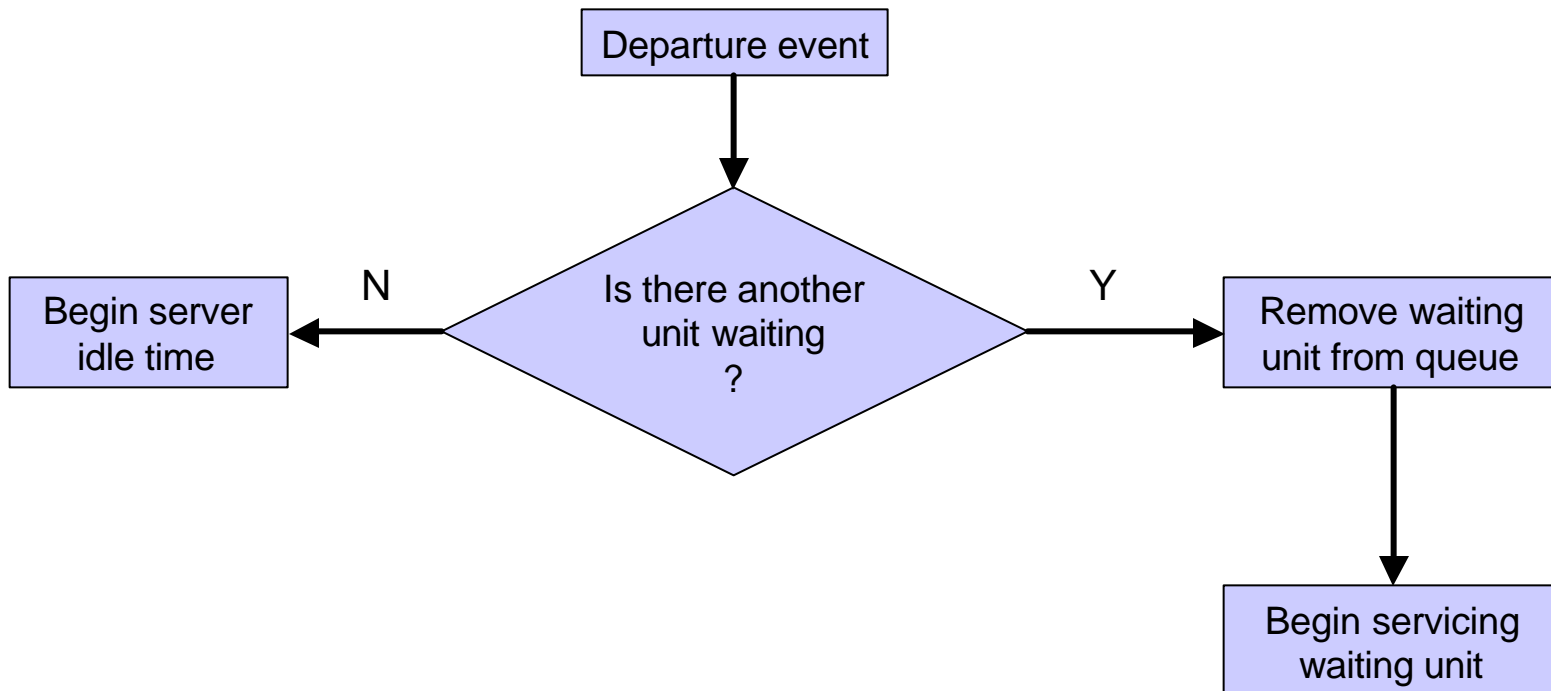
# Queuing system simulation



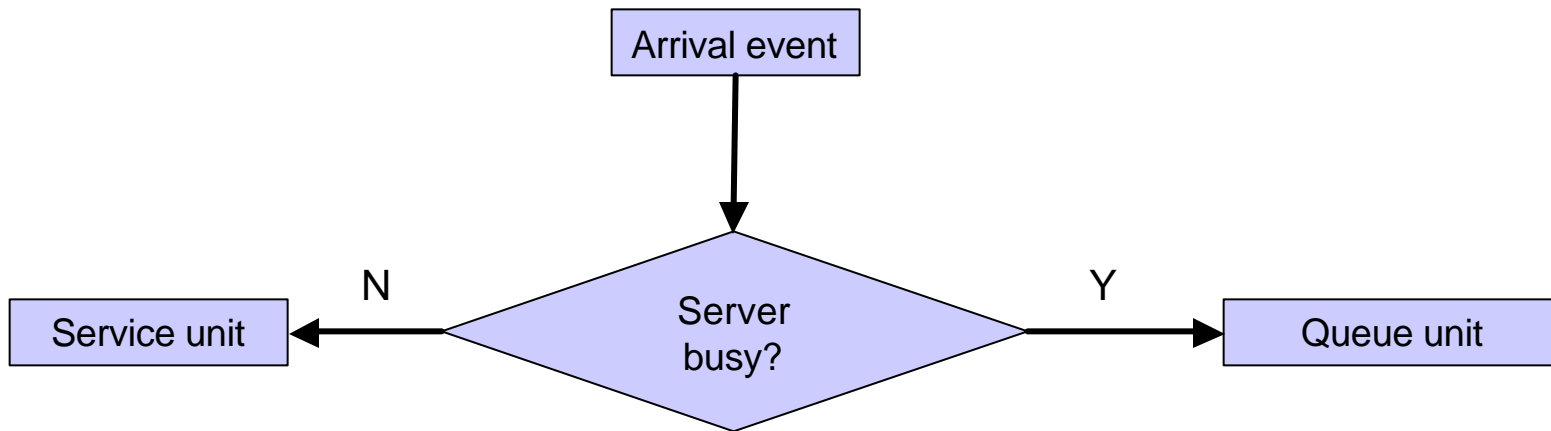
# Queuing Simulation Terminology

- System state: The number of units (e.g., waiting callers) in the system and the status of the server (busy or idle)
- Event: something that causes a change in state of the system (e.g., a call arrival or departure)
- Simulation clock: tracks simulated system time.

# Service-just-completed flow



# Unit-entering-system flow



# Actions upon arrival

		Queue status	
		Not empty	Empty
Server Status	Busy	enqueue	enqueue
	Idle	Impossible	Enter service

# Server outcomes after service completion

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

# Random numbers in simulations

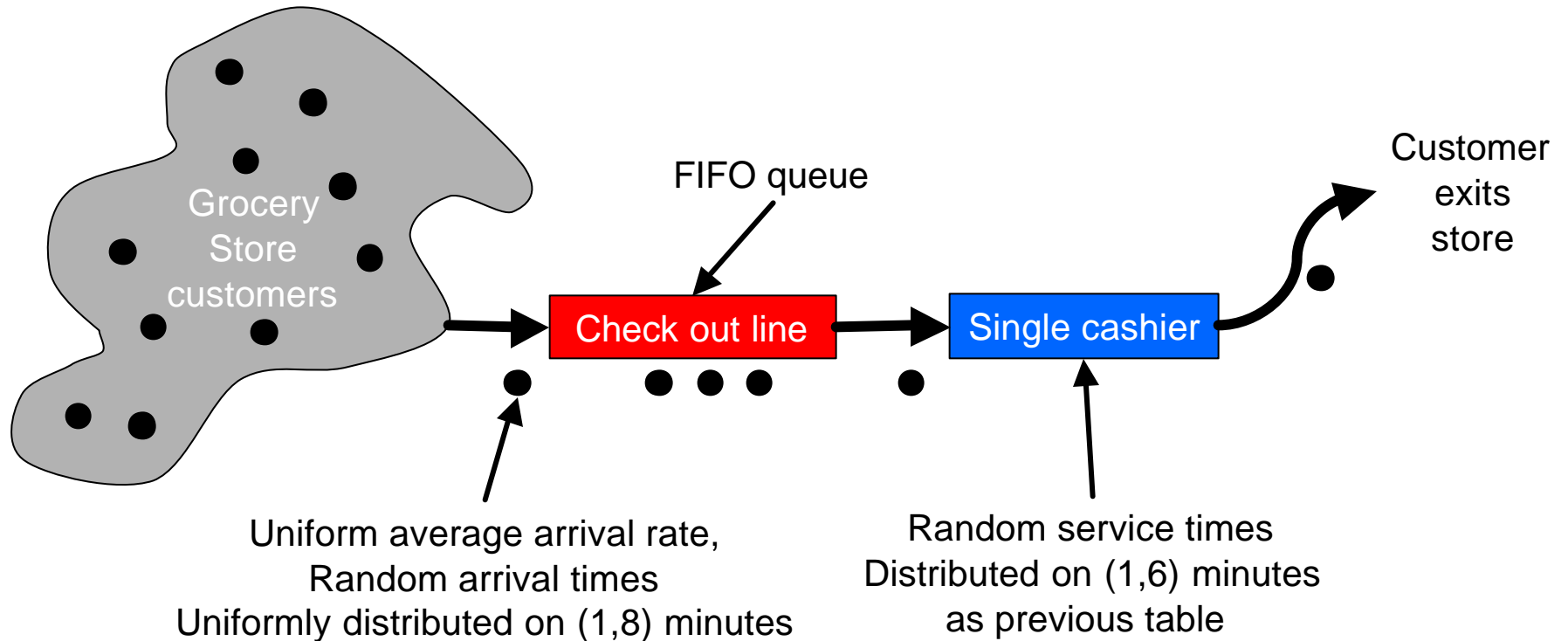
- Random numbers are needed to simulate:
  - Service times
  - Inter-arrival times
- Most often, it is easiest to work with random numbers uniformly generated on the interval  $(0,1)$ . These can be used to create:
  - Random digits (by quantizing)
  - Random numbers on other intervals (by scaling)
  - Other distributions (e.g., Poisson, Exponential, Normal)
- Truly random numbers are very difficult to create.
  - Pseudo-random numbers normally have to suffice
  - Generate random numbers from tables of random digits. (harder to automate)

# Generating a simple distribution

Service time (minutes)	Probability	Cumulative Probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

```
int service_time(void)
{
    r=rand()/RAND_MAX;    /* pseudorandom r on (0,1) */
    if(r<.1) return(1);
    else if(r<.3) return(2);
    else if(r<.6) return(3);
    else if(r<.85) return(4);
    else if(r<.95) return(5);
    else return(6);
}
```

# Simulating a single server queue



Simulate arrival, service for 20 customers

# Time-Between-Arrivals

Customer	Time between arrivals (minutes)		Customer	Time between arrivals (minutes)
1	-		11	1
2	8		12	1
3	6		13	5
4	1		14	6
5	8		15	3
6	3		16	8
7	8		17	1
8	7		18	2
9	2		19	4
10	3		20	5

Arrival times have been generated randomly, as described in text

# Service Times

Customer	Service Time (minutes)		Customer	Service Time (minutes)
1	4		11	3
2	1		12	5
3	4		13	4
4	3		14	1
5	2		15	5
6	4		16	4
7	5		17	3
8	4		18	3
9	5		19	2
10	3		20	3

Arrival times have been generated randomly, as described in text

# Simulation Table for Queuing Problem

Customer	Time since last arrival	Arrival Time	Service Time	Time Service Begins	Time in Queue	Time Service Ends	Customer Time in System	Idle Time of Server
1	-	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
total			68		56		124	18

# Queue Statistics

- Average waiting time:  
(Total customer waiting time)/(total number of customers) =  $56/20 = 2.8$  minutes
- Probability customer has to wait in queue:  
 $P(\text{wait}) = (\text{number of customers who wait})/(\text{total number of customers}) = 13/20 = 0.65$
- Fraction of idle time of server:  
 $P(\text{idle}) = (\text{total idle time})/(\text{total run time of simulation}) = 18/86 = 0.21$
- Average service time:  
(total service time)/(total number of customers) =  $68/20 = 3.4$  minutes
- Expected service time:  
 $E(S) = \sum s \cdot p(s) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.25) + 5(0.1) + 6(0.05) = 3.2$  minutes

Why do the average service time and the expected service time differ?

# Queue Statistics

- Average time between arrivals:

$$\begin{aligned} & (\text{sum of all times between arrivals}) / (\text{number of arrivals} - 1) = 82/19 \\ & = 4.3 \text{ minutes} \end{aligned}$$

- Expected time between arrivals:

$$E(A) = (1+8)/2 = 4.5 \text{ minutes}$$

Why do average and expected time between arrivals differ?

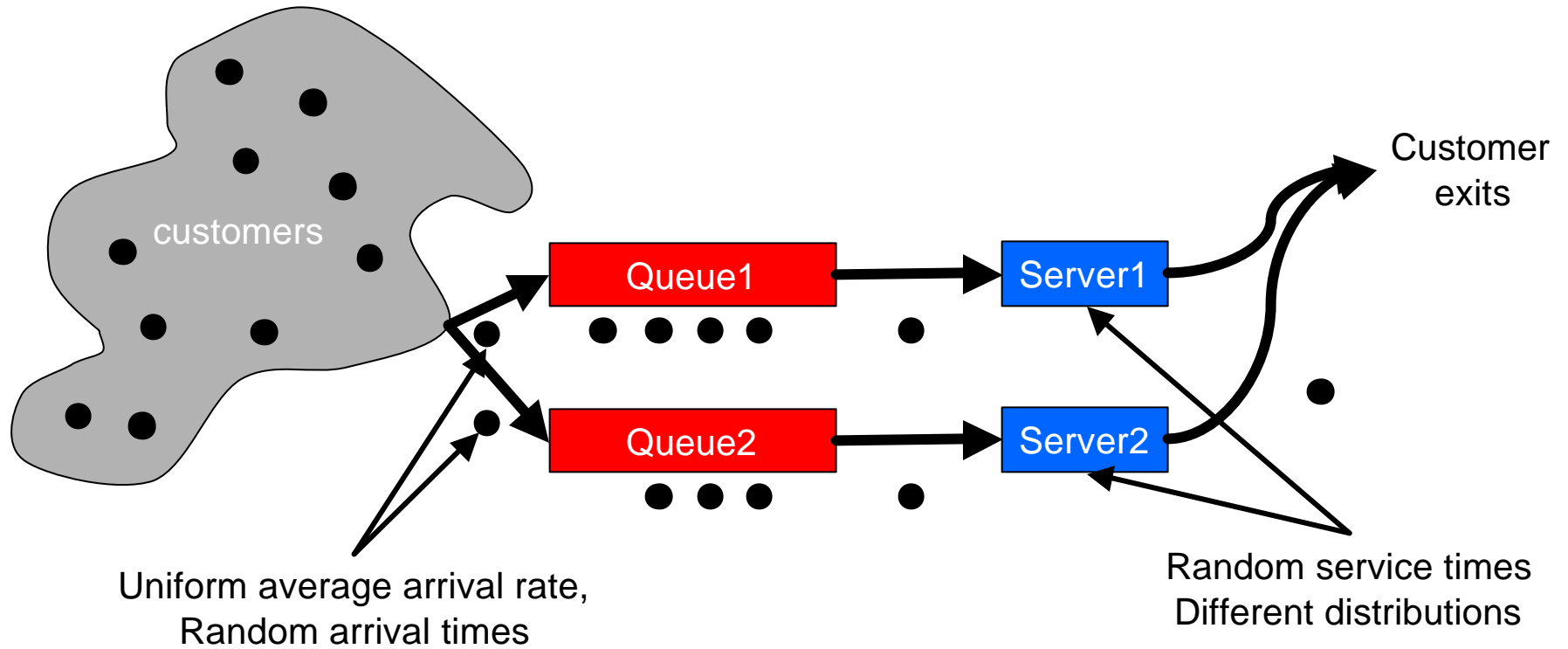
- Average waiting time for those who wait:

$$\begin{aligned} & (\text{total time customers wait in queue}) / (\text{total customers who wait}) \\ & = 56/13 = 4.3 \text{ minutes} \end{aligned}$$

- Average time spent in system:

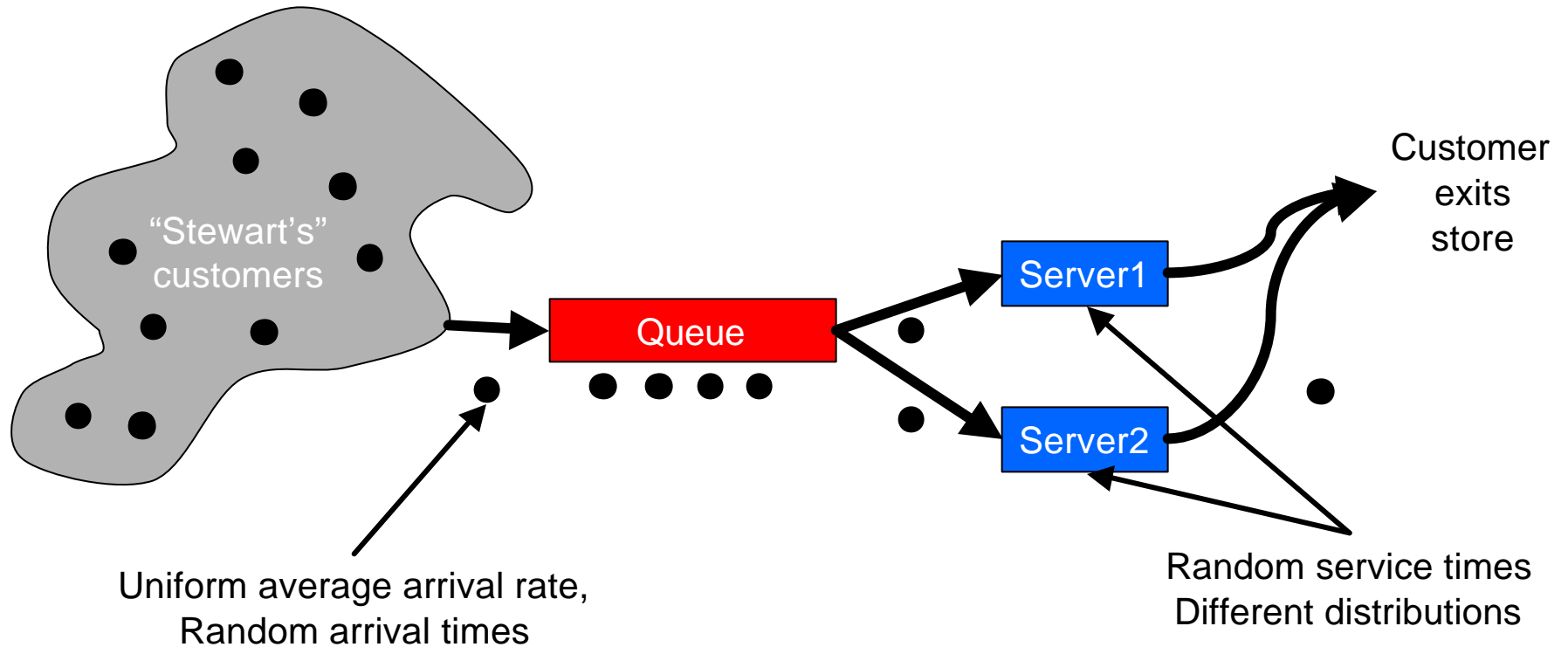
$$\begin{aligned} & (\text{total time customers spend in system}) / (\text{total number of customers}) \\ & = 124/20 = 6.2 \text{ minutes} \\ & = (\text{average time in queue}) + (\text{average time in service}) = 2.8 + 3.4 = 6.2 \text{ minutes} \end{aligned}$$

# Multi-server Queues



# Multi-server Queues

## “Able-Baker car hops”



Which “performs better”  
the single queue/multiple server  
or multiple queue/multiple server?

# Inter-arrival Distribution of Customers

Time between arrivals (minutes)	Probability	Cumulative probability
1	0.25	0.25
2	0.40	0.65
3	0.20	0.85
4	0.15	1.00

# Service time distribution

Able

Service Time (minutes)	Probability	Cumulative probability
2	0.30	0.30
3	0.28	0.58
4	0.25	0.83
5	0.17	1.00

Baker

Service Time (minutes)	Probability	Cumulative probability
3	0.35	0.35
4	0.25	0.60
5	0.20	0.80
6	0.20	1.00

Simplifying assumption: Able gets customer if idle, else customer goes to next available server.

# Able-Baker Simulation Table

Customer	Time between arrivals	T.O.A	ABLE			BAKER			Time in queue
			Time service begins	Service time	Time service ends	Time service begins	Service time	Time service ends	
1	-	0	0	5	5				0
2	2	2				2	3	5	0
3	4	6	6	3	9				0
4	4	10	10	5	15				0
5	2	12				12	6	18	0
6	2	14	15	3	18				1
7	3	17	18	2	20				1
8	3	20	20	4	24				0
9	3	23				23	4	27	0
10	1	24	24	3	27				0
11	2	26	27	3	30				1
12	2	28				28	4	32	0
13	2	30	30	5	35				0
14	1	31				32	3	35	2
15	2	33	35	4	39				0
16	2	35				35	4	39	2
17	2	37	39	4	43				0
18	3	40				40	5	45	1
19	2	42	43	2	45				1
20	2	44	45	4	49				0
21	4	48				48	3	51	0
22	1	49	49	3	52				0
23	2	51				51	5	56	0
24	3	54	54	3	57				0
25	1	55				56	6	62	1
26	4	59	59						0

# Modeling/Simulation Project Ideas

## - a few *possible* topics

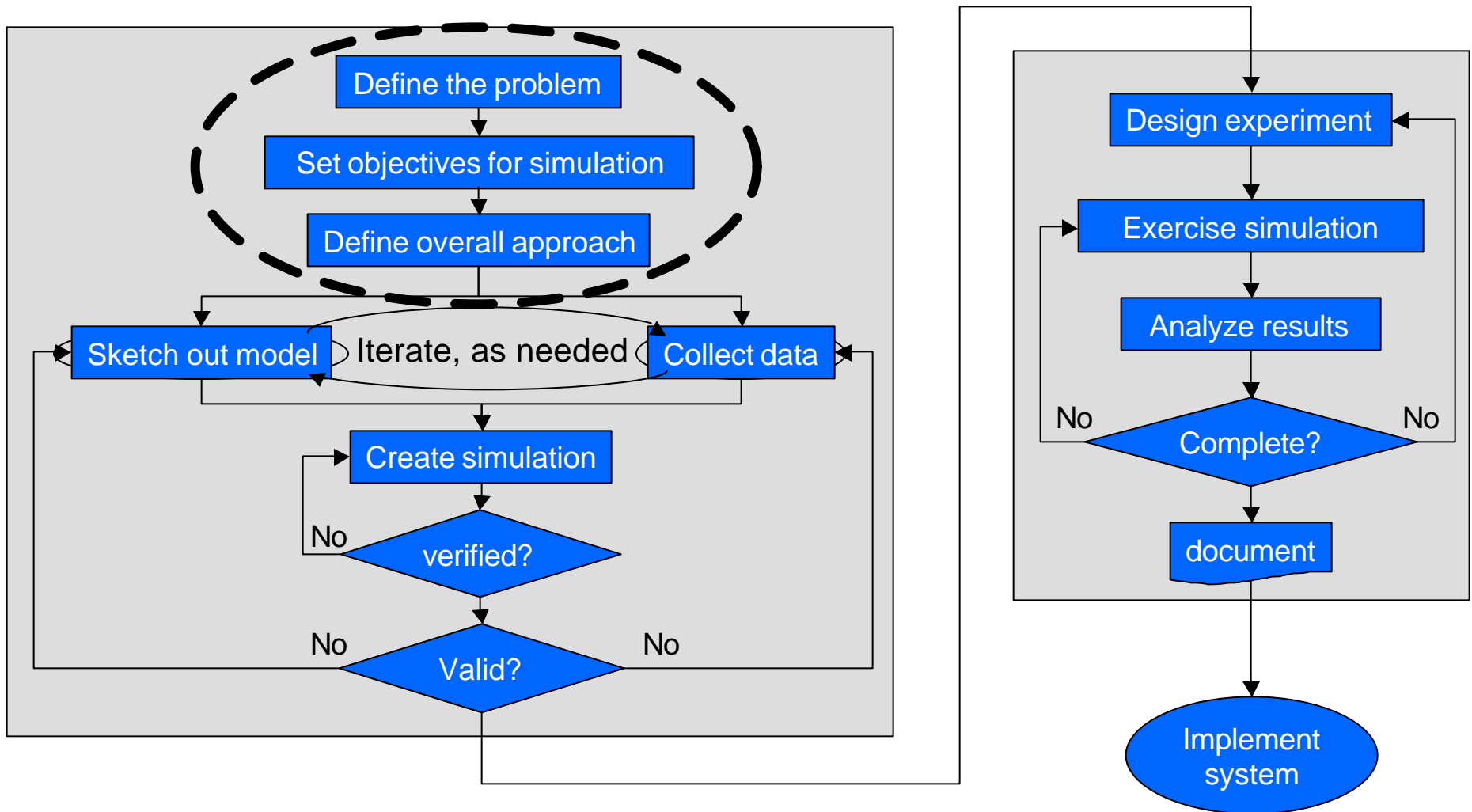
- Spread of disease in a human population (e.g., HIV, influenza, smallpox, measles)
- Spread of a computer virus/worm/trojan horse (e.g., Word macroviruses)
- Average classroom utilization at Stevens
- Data throughput on the Stevens network
- Average waiting time at the Burchard Auditorium Java City
- Average waiting time at Howe Center elevators.  $\{f(\text{floor}, \text{time}, \text{direction})\}$   
(There **has** to be a better algorithm than the elevators currently use - propose one and simulate it)
- Scheduling of PATH, NJ Transit trains
- Dynamic behavior of power grid
- Random vs. scale-free networks (Scientific American, May 2003)
- Parking spot availability in Hoboken
- Scheduling of traffic lights on Washington Street
- Traffic patterns in a large communications network

Topic *must* deal with a ***DYNAMIC, STOCHASTIC, DISCRETE-EVENT*** problem:

- system behavior must be variable in time
- there must be a random or highly complex aspect of system
- state changing events must occur at discrete points of time, not continuously

Project must be related to a real system you can gather data on

# Steps in a simulation study



# Homework 2

A: Due 2/9: Chapter 2, page 61, 62 - problems 26, 28

B: Due 2/17 (two weeks):

Research the possible simulation project ideas.

Formulate a simulation plan and present a proposal (1-2 pages)

- Project title
- What you will study
- Objectives for the study
- Overall approach
- How you will go about gathering data
- Project group members
  - Group members must sign proposal
  - Group members must be enrolled in EE/PEP-345
  - Group size should be 3-5 members.