

EE345 Modeling and Simulation
 Spring 2004
 HW4
 Chapter 5, problems 14, 18, 20

Problem 14: The lifetime, in years, of a satellite placed in orbit is given by:

$$f(x) := .4 \cdot e^{-.4x} \quad \text{for } x \geq 0, 0 \text{ otherwise}$$

- (a) what is the probability that the satellite is still alive after 5 years
 (b) what is the probability that the satellite dies between 3 and 6 years from the time it is placed in orbit?

(a) X is the lifetime of the satellite. X is exponentially distributed.
 The cumulative distribution of X is:

$$F(x) := 1 - e^{-.4x} \quad \text{for } x \geq 0$$

$$P(X \geq 5) = 1 - F(5)$$

$$1 - F(5) = 0.135$$

(b) the probability that the satellite dies between 3 and 6 years is:

$$P(3 \leq X \leq 6) = F(6) - F(3)$$

$$F(6) - F(3) = 0.21$$

Problem 18: Accidents at an industrial site occur one at a time, independently, and completely at random at a mean rate of one per week. What is the probability that no accidents occur in the next three weeks?

X is the number of accidents in a week

Consider this to be a Poisson arrival process with mean α

$$\alpha := 1$$

$$p(n) := \frac{e^{-\alpha} \alpha^n}{n!}$$

$$p(0) = 0.368$$

Since the weeks are independent, the probability of no accidents in 3 weeks is the probability that no events will occur in each week.

$$P_{\text{no_accidents_3_weeks}} := p(0)^3 \quad P_{\text{no_accidents_3_weeks}} = 0.05$$

Problem 20: Suppose that a Die-Hardly Ever battery has an exponential time-to-failure distribution with a mean of 48 months. After 60 months, the battery is still operating.

- (a) what is the probability that the battery is going to die in the next 12 months?
 (b) what is the probability that the battery dies in an odd year of its life?
 (c) if the battery is operating at 60 months, compute the expected additional months of life

Since the time-to-failure distribution is exponential, the cumulative distribution is:

$$F(x) := 1 - e^{-\frac{x}{48}}$$

(a) The probability of failure between months 60 and 72 is the same as the probability that the battery will fail in 12 months, since the exponential distribution is memory-less.

$$F(12) = 0.221$$

(b) The probability that the battery fails in an odd year Y is:

$$P(Y = \text{odd}) = (1 - e^{-12/48}) + [(1 - e^{-36/48}) - (1 - e^{-24/48})] + \dots$$

$$P(Y = \text{odd}) = 1 - e^{-.25} + e^{-.5} - e^{-.75} + \dots$$

Which means $P(\text{year 1}) + [P(\text{year 3 but not year 2})] + \dots$

Likewise, the probability of failure in an even year is:

$$P(Y = \text{even}) = [(1 - e^{-24/48}) - (1 - e^{-12/48})] + [(1 - e^{-48/48}) - (1 - e^{-36/48})] \dots$$

$$P(Y = \text{even}) = e^{-.25} - e^{-.5} + e^{-.75} - e^{-1.0} + \dots$$

which means $[P(\text{year 2 but not year 1}) + P(\text{year 4 but not year 3})]$

so:

$$P(Y = \text{even}) = e^{-.25} P(Y = \text{odd})$$

since:

$$1 - P(Y = \text{even}) = P(Y = \text{odd})$$

therefore,

$$1 - e^{-.25} P(Y = \text{odd}) = P(Y = \text{odd})$$

and,

$$P(Y = \text{odd}) = \frac{1}{1 + e^{-.25}}$$

$$P_{\text{odd}} := \frac{1}{1 + e^{-.25}} \quad P_{\text{odd}} = 0.562$$

(c) - the exponential process is memory-less. The expected additional months of life is still 48 months.