Example 8.1 - Histogram of exponentially distributed random numbers using Inverse Transform Technique

$N := 200$

$i := 0..\, N - 1 \qquad \lambda := 1$

$R := \mathrm{runif}(N, 0, 1)$

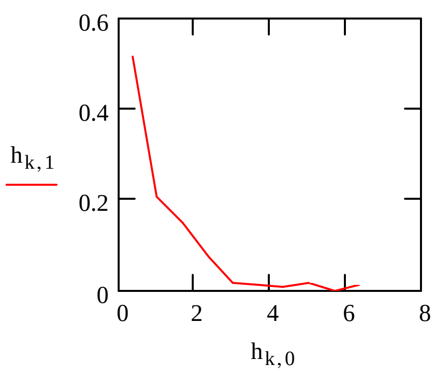
$$X := -\left(\frac{1}{\lambda}\right)\cdot \ln(R)$$

$M := 10$

$h := \mathrm{histogram}(M, X)$

$k := 0..\, M - 1$

$$h_{k,1} := \frac{h_{k,1}}{N}$$



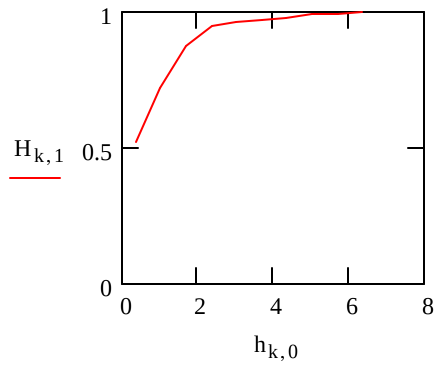
Find c.d.f.

$H := h$

$H_{0,1} := h_{0,1}$

$k2 := 1..\, M - 1$

$H_{k2,1} := H_{k2-1,1} + h_{k2,1}$


1
0.5
0
0
2
4
6
8
$H_{k,1}$
$h_{k,0}$

Weibull Distribution

$N := 2000$

$i := 0..\ N - 1$

$\alpha := 1$

$R := \text{runif}(N, 0, 1)$

$\beta := 1.5$

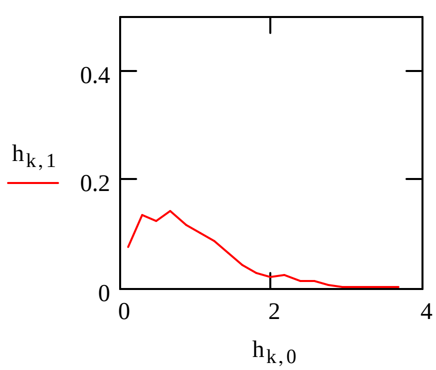
$X := \alpha \cdot (-\ln(R))^{\frac{1}{\beta}}$

$M := 20$

$h := \text{histogram}(M, X)$

$k := 0..\ M - 1$

$h_{k,1} := \dfrac{h_{k,1}}{N}$



Triangular Distribution

$N := 20000$

$i := 0..\, N - 1$

$R := \mathrm{runif}(N, 0, 1)$

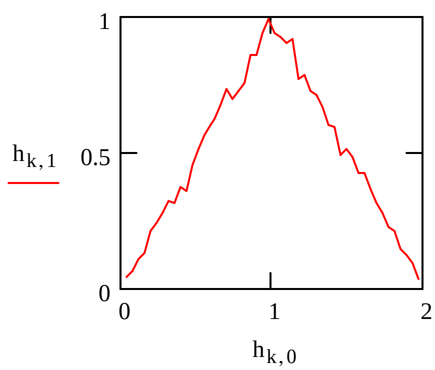
$X_i := \mathrm{if}\left[R_i \le .5, \sqrt{2 \cdot R_i}, 2 - \sqrt{2 \cdot \left(1 - R_i\right)}\right]$

$M := 50$

$h := \mathrm{histogram}(M, X)$

$k := 0..\, M - 1$

$$h_{k,1} := \frac{h_{k,1} \cdot \frac{M}{2}}{N}$$



Normal Distribution using Direct Transform

$N := 1000$

$i := 0..\ N - 1$

$R_1 := \mathrm{runif}(N, 0, 1)$

$R_2 := \mathrm{runif}(N, 0, 1)$

$Z_{1_i} := \sqrt{-2 \cdot \ln\left(R_{1_i}\right)} \cdot \cos\left(2 \cdot \pi \cdot R_{2_i}\right)$ $\qquad \mu := 2$

$Z_{2_i} := \sqrt{-2 \cdot \ln\left(R_{1_i}\right)} \cdot \sin\left(2 \cdot \pi \cdot R_{2_i}\right)$ $\qquad \sigma := 2.5 \qquad \sigma^2 = 6.25$

$X_1 := \mu + \sigma \cdot Z_1$

$X_2 := \mu + \sigma \cdot Z_2$

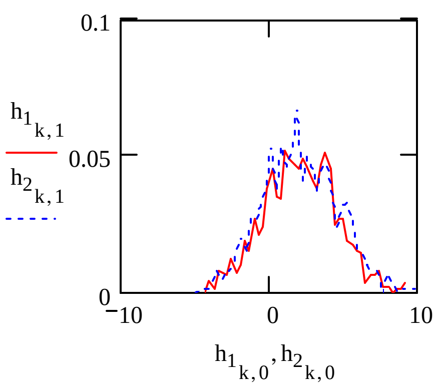
$M := 50$

$h_1 := \mathrm{histogram}\left(M, X_1\right)$

$h_2 := \mathrm{histogram}\left(M, X_2\right)$

$k := 0..\ M - 1$

$h_{1_{k,1}} := \frac{h_{1_{k,1}}}{N} \qquad h_{2_{k,1}} := \frac{h_{2_{k,1}}}{N}$



$\mathrm{mean}\left(X_1\right) = 2.029$

$\mathrm{mean}\left(X_2\right) = 2.05$

$\mathrm{var}\left(X_1\right) = 5.842$

$\mathrm{var}\left(X_2\right) = 6.294$

Example 8.11 - Bus with Poisson arrival process, average number of arrivals/hour = 4

$\alpha_{bus} := 4$

$e^{-\alpha_{bus}} = 0.018$

$$N(\alpha) := \left| \begin{array}{l} n \leftarrow 0 \\ P \leftarrow 1 \\ m \leftarrow e^{-\alpha} \\ \text{while } P \geq m \\ \quad \left| \begin{array}{l} P \leftarrow P \cdot \text{rnd}(1) \\ n \leftarrow n + 1 \end{array} \right. \\ \text{return } n \end{array} \right.$$

How many arrivals per hour during a given 8 hour period?

$i := 0..\,7$

$\text{Arrivals}_i := N(\alpha_{bus})$

$$\text{Arrivals} = \begin{pmatrix} 1 \\ 4 \\ 4 \\ 10 \\ 3 \\ 8 \\ 2 \\ 6 \end{pmatrix} \qquad \text{mean(Arrivals)} = 4.75$$