

Homework 5 solution:

Generate a set of exponentially distributed random numbers with mean 3.

Note: the `rexp()` function uses a "rate" parameter which is the reciprocal of the mean

`m := 3`

`N := 100`

`x := rexp(N, 1/m)`

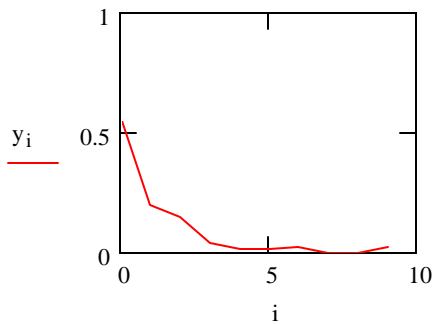
`mean(x) = 3.105`

Create a histogram (probability density function) of the distribution with 10 bins

`bins := 10`

`y := hist(bins, x) / N`

`i := 0..bins - 1`



I didn't ask for a histogram, but it helps verify that the distribution looks like we expected it to.

Generate a normal distribution with mean 1 and variance 1

$$\mu := 1 \quad \sigma := \sqrt{1}$$

$$x := \text{rnorm}(N, \mu, \sigma)$$

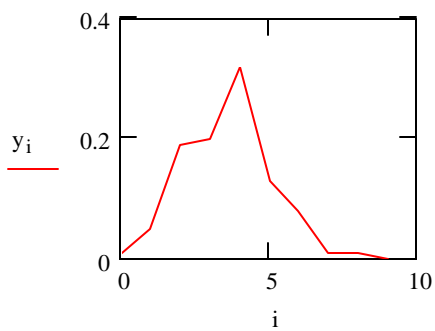
$$\text{mean}(x) = 0.888$$

$$\text{stdev}(x)^2 = 0.993$$

Create a histogram (probability density function) of the distribution with 10 bins

$$y := \frac{\text{hist}(\text{bins}, x)}{N}$$

$$i := 0.. \text{bins} - 1$$



What percentage of the time does the (experimentally generated) normal distribution exceed the mean by 2 standard deviations? What percentage is expected?

Count the events that exceed  $\mu + 2\sigma$ :

$$\text{threshold} := \mu + 2 \cdot \sigma$$

$$\text{threshold} = 3$$

$$i := 0.. N - 1$$

$$\text{exceed} := x > \text{threshold}$$

$$\text{percent} := 100 \cdot \frac{\sum_i \text{exceed}_i}{100}$$

$$\text{percent} = 2$$

What percentage is expected? For this, we have to find the integral of a normal p.d.f. from  $\mu+2\sigma$  to infinity. Mathcad provides cnorm for the cumulative normal distribution from -infinity to x, We just have to subtract from 1 (cnorm is normalized for  $\mu=0, \sigma=1$ )

expected\_percentage := 100·(1 – cnorm(2))

expected\_percentage = 2.275