

Midterm for CpE/EE 345 – Modeling and Simulation Stevens Institute of Technology Fall 2002	Name:
Pledge:	

October 21, 2002

The midterm is open book/open notes. Total value is 30 points. All questions are equally weighted. Do any 15 of 18 questions. Do more than 15 for extra credit. Some questions could be answered in more than one way. Only one answer is required, but extra credit will be given for identifying and explaining alternative answers.

Return the exam question pages with your answers.

1. You have been assigned the task of simulating a complex system and you have collected data about an input process to the system. You have constructed an empirical probability density function with the data listed below. How would you create values to represent this input process in your simulation?

Value	Probability	Cumulative probability
0.5	0.517	0.517
1.0	0.256	0.773
1.5	0.115	0.888
2.0	0.049	0.937
2.5	0.033	0.970
3.0	0.016	0.986
3.5	0.004	0.990
4.0	0.005	0.995
4.5	0.002	0.997
5.0	0.003	1.000

Answer 1: Generate a uniformly distributed random number between 0 and 1. Use that number to lookup values in the c.d.f. column. Use the corresponding number in the value column as a value for the R.V.

Answer 2: The distribution resembles an exponential distribution. Find a mean value that approximates the empirical data and use a exponential R.V. generator to generate the random inputs.

2. What is the expected value of the random process described in Question 1?

$\text{Sum}(\text{value} \cdot p(\text{value})) = .973$

3. What is meant by the "System State" in a simulation.

The system state is the information needed to fully describe the system at any point in time. It is the set of values of all state variables in the system, the state variables being the attributes of all the entities (or objects of interest) in the system.

4. Bits are transmitted through a communications system correctly 99% of the time. Bit errors are independent. What is the probability that a block of 100 bits is transmitted with no more than 3 bit errors?

Consider this as a Bernoulli trial with a sample size of 100 and  $p=.01$ .

$$P(X \leq 3) = \sum_{x=0}^3 \binom{100}{x} (10^{-2})^x (.99)^{100-x}$$

where

$$\binom{100}{x} = \frac{100!}{x!(100-x)!}$$

$$= (.99)^{100} + (100) \cdot (10^{-2}) \cdot (.99)^{99} + (100)(99) \cdot (10^{-2})^2 \cdot (.99)^{98} + (100)(99)(98) \cdot (10^{-2})^3 \cdot (.99)^{97}$$

$$= .982$$

5. A factory assembly line produces subassemblies at an average rate of 6 units per hour with a distribution in completion times that can be modeled as a Poisson process. The assemblies are sent to two testing stations. The first testing station randomly receives 30% of the assemblies; the second testing station receives the rest. One tester who can test an average of two assemblies per hour staffs the first testing station. Two testers staff the second testing station. One can test 3 assemblies per hour; the second can test one assembly per hour. What are the average numbers of assemblies waiting for each testing station?

The poisson arrival process can be split into two processes, based on the random splitting property. The first test station sees an arrival rate of  $6 \cdot .3 = 1.8$ /hour. The second station sees  $6 \cdot .7 = 4.2$  assemblies per hour. Since the second station has two servers with capacities of 1 and 3, the total capacity of the second station is 4/hour, which is less than the arrival rate. Thus, the average number of assemblies waiting at the second testing station grows without bound. For the first station, by Little's equation, the average number of assemblies waiting is the arrival rate time the average service time per item. This is:  $1.8 \cdot (1/2)$  or .9.

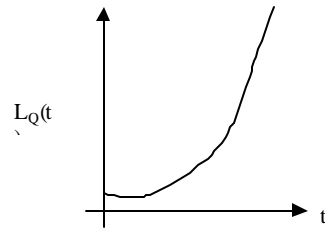
6. A certain college has 5 printers in the computer room for the use of the college's 1500 undergraduate and 2000 graduate students. Print jobs arrive uniformly during the day and have an exponentially distributed page count. They are spooled on a single hard disk while waiting for the first printer to be available. In the standard notation of queuing systems, how would this queuing system be characterized? (e.g., as an M/M/1 queue? Or some other type?)

G/M/5/N/3500: General arrival, exponential service (assuming service time ~ length), 5 servers, a limit of N jobs in the queue (it has to be finite, based on the hard disk) and 3500 students in the population (again, finite). The hard disk is not a server, the printers are the servers, so  $c=5$ , not 1.

7. Two similar terms used in the steps of a simulation study are "verification" and "validation." One of them refers to the debugging of the simulation code itself. Which one refers to the process of insuring that the model is a correct representation of the system?

Validation

8. You observe the behavior graphed below in a queuing system. What observation can you make about  $r$ ?



The queue length is growing, apparently without bound. The server utilization,  $r$ , is probably greater than 1.

9. Assuming an exponentially distributed service time, which queuing discipline is likely to have the shortest average waiting time: FIFO, LIFO, Priority, or SPT?

SPT (Shortest Processing First) will most often have the shortest average waiting time, since the server can process a large number of short jobs first. The longer jobs move to the rear of the queue and, although they will have longer waiting times, there are far fewer of them for exponentially distributed service times.

10. An arrival event occurs in a queuing system when the queue is not empty. What observation can you make about the status of the server?

It must be busy. If it were idle, it would have taken a waiting customer from the queue.

11. Under what circumstances might a simulation report  $L_Q$  exceeding  $L$ ?

None, if it is working properly.  $L$  records the long term average number of customers in the system,  $L_Q$  records the long term average number of customers waiting in queue. Since every customer in the queue is also in the system,  $L_Q$  could never be greater than  $L$ . If a simulation is reporting this circumstance, there must be a bug in the simulation, miscounting customers.

12. What are two events that can cause a change in a queuing system's state?

Customer arrival, service completion (customer departure)

13. We are simulating a packet switching system, where a linked list is being used to simulate the input buffer that stores one character per list record. The list contents at one point in time are shown in the table below. What is the data stored in the buffer if the "head" pointer value is 2?

Array position	Record	Next
1	"space"	5
2	C	7
3	C	11
4	E	3
5	I	6
6	N	2
7	O	9
8	R	4
9	R	8
10	S	4
11	T	11

CORRECT

14. Why is it important to maintain order in the FEL?

The Future Event List is sorted in the time order of future events. In this way, the next event (the imminent event) is always the first item in the list, avoiding the need to search the entire list each time it is processed.

15. The arrival of customers at a the Podunk, New Jersey motor vehicle inspection station can be modeled as a Poisson process with an arrival rate of 12 per hour. They randomly choose one of three lanes, but cannot leave the lane or change lanes once they make their decision. The service times associated with each lane are exponentially distributed with mean values of 5, 10 and 15 minutes. As they exit the inspection station, all lanes merge into one. What is the arrival rate of vehicles in the single exit lane?

Using the random splitting property, the 12 per hour arrivals are split into three 4 per hour streams. For the lanes that have service times of 5 and 10 minutes, the service rate exceeds the arrival rate, so the queue is bounded and the average departure rate equals the average arrival rate (4 per hour). The lane with an average service time of 15 minutes is operating at 100% utilization, so this queue is not bounded. The departure rate for this queue will be limited by the service rate, which is 4 per hour. Strictly speaking, this is the upper bound of the departure rate, which may be slightly less due to variance in the service times and the resulting queue growth. By the Poisson pooling property, the total departure rate is the sum of the departure rates or (4)+(4)+(4-). "12 per hour" or "slightly less than 12 per hour" would be acceptable answers.

16. The variance of the Weibull distribution is given by:

$$V(X) = a^2 \left[ \Gamma\left(\frac{2}{b} + 1\right) - \left[ \Gamma\left(\frac{1}{b} + 1\right) \right]^2 \right]$$

where

$$\Gamma(b) = \int_0^{\infty} x^{b-1} e^{-x} dx$$

use this to find the variance of an exponential distribution with a mean of 2.

You could observe that the variance of the exponential is the square of the mean, but you could also recognize that the exponential is a special case of the Weibull distribution with  $b=1$ . In this case, the variance of the distribution is

$$V(X) = a^2 \left[ \Gamma\left(\frac{2}{1} + 1\right) - \left[ \Gamma\left(\frac{1}{1} + 1\right) \right]^2 \right] =$$

$$a^2 \left[ \Gamma(3) - [\Gamma(2)]^2 \right] = a^2 \left[ (3-1)! - [(2-1)!]^2 \right] =$$

$$a^2 [2 - 1] = a^2$$

and since  $a=2$  is the mean of the exponential, the variance is 4.

17. The customer arrival process at a particular store is exponentially distributed with a mean arrival rate of 3 customers per hour. No customers have arrived in the last 5 hours. How many customers are expected to arrive in the next hour?

3, the expected number of future arrivals is independent of the past arrivals and, in particular, the time since the last arrival.

18. At  $t=t_1$ , the Future Event List for a system being simulated was:  $\{(1,15.5), (2,18.3), (1,21.5)\}$ , where a Type 1 event is a customer arrival and a Type 2 event is service completion by the single server in the system. If a customer is scheduled to arrive at  $t=20$ , what is the FEL just after  $t=18.3$ ?

$\{(1,20), (1,21.5)\}$