Chapter 5, p.198 problems 15 and 25

15) A mainframe computer crashes in accordance with a Poisson process with a mean rate of one crash every 36 hours. Determine the probability that the next crash will occur between 24 and 48 hours after the last crash.

Let X represent the number of hours until a crash occurs. The distribution of time between crashes is exponentially distributed. So, if F(X) is the cumulative distribution of an exponential inter-arrival process with mean arrival time of 36 hours, the probability of a crash occurring between 24 and 48 hours after the last crash is:

$$F(48)-F(24)=\left(1-e^{48\bullet\frac{1}{36}}\right)-\left(1-e^{24\bullet\frac{1}{36}}\right)=e^{-24/36}-e^{-48/36}=.513-.264=.249$$

so, the probability of a crash in the interval specified is .249.

25) The time intervals between dial-up connections to an Internet service provider are exponentially distributed with a mean of 15 seconds. Find the probability that the third dial-up connection occurs after 30 seconds have elapsed.

This problem is defining the probability of combinations of exponential arrival processes, so, just as trunk usage in a telephone network, the Erlang distribution is pertinent.
In this case, the parameters of the Erlang distribution are:
Kq = 1/15 (arrival rate) and *X* = 30 (interval)
The probability that the third connection occurs within 30 seconds is:

$$F(30)=1-\sum_{i=0}^{2}\frac{e^{-\frac{1}{15}(30)}\left(\frac{1}{15}30\right)^{i}}{i!}=.323$$

Since the problem asks the probability that the third connection occurs after 30 seconds have elapsed, the complement of F(30) is the proper answer, so the probability is 1-.323 or .677.