Chapter 6, p.247,248 problems 1 and 10

1) A tool crib has exponential interarrival and service times, and it serves a very large group of mechanics. The mean time between arrivals is 4 minutes. It takes 3 minutes on the average for a tool-crib attendant to service a mechanic. The attendant is paid $10 per hour and the mechanic is paid $15 per hour. Would it be advisable to have a second tool-crib attendant?.

The tool crib is modeled as a M/M/c queue with l = ¼, m = 1/3 and c=1 or 2. Given that the attendants are paid $10 per hour and the mechanics are paid $15 per hour, consider that the mechanics are generating unpaid overhead while waiting in the queue, while the attendants are 100% overhead, since they generate no direct revenue.

$$\text{mean cost per hour} = \$10c + \$15L$$

where c is the number of attendants and L is the average number of mechanics in the system.

For 1 attendant, this is an *M/M/1* queue. (c=1, $r = l/m$ =.75)
L= ρ/(1−ρ) = 3 mechanics.
Cost per hour is $10(1) + $15(3) = $55/hour

For 2 attendants, this is an *M/M/2* queue. (c=2, $r = l/cm$ = .375)

$$L = cr + \left[(cr)^{c+1} P_0 \right] / [c(c!)(1-r)^2] = .8727$$

$$where$$

$$P_0 = \left\{ \left[\sum_{n=0}^{c-1} (cr)^n / n! \right] + \left[(cr)^c (1/c!)(1/(1-r)) \right] \right\}^{-1} = .4545$$

so the mean cost per hour is
$10(2) + $15(.8727) = $33.09 per hour.
It is cheaper to have a second attendant to avoid the cost of waiting mechanics.

10) Arrivals to a self-service gasoline pump occur in a Poisson fashion at a rate of 12 per hour. Service time has a distribution which averages 4 minutes with a standard deviation of 1 1/3 minutes. What is the expected number of vehicles in the system?

The gasoline pump is modeled by a M/G/1 queue with l = 12/hour, m = 15/hour, and $s^2 = 1.33^2\ \text{min}^2 = 0.0222^2\ \text{hour}^2$

$r = l/m = .8$

$$L = r + \left[r^2 \left(1 + s^2 m^2\right) \right] \Big/ \left[2\left(1 - r\right) \right] = 2.5778 \text{ vehicles}$$