

Midterm for CpE/EE 345 – Modeling and Simulation Stevens Institute of Technology Spring 2003	Name:
Pledge:	

March 3, 2003

The midterm is open book/open notes. Total value is 30 points. All questions are equally weighted. **Do all of the first 10 questions. Do any 2 of the last 4 questions. Do more than 2 for extra credit.** Some questions could be answered in more than one way. Only one answer is required, but extra credit will be given for identifying and explaining alternative answers.

Return the exam question pages with your answers.

1. The customer arrival process at a particular store is exponentially distributed with a mean arrival rate of 8 customers per hour. 24 customers have arrived in the last 2 hours. How many customers are expected to arrive in the next 15 minutes?

[1A3] .9

[4A2] .1

2, the expected number of future arrivals is independent of the past arrivals and, in particular, the time since the last arrival. The number of arrivals within an interval depends only on the length of the interval.

2. What is meant by the "System State" in a simulation? What can change the system state in a queuing system?

[3B3]

(2/3rds of the value of the question is for the first part, 1/3 for the second part). The system state is the information needed to fully describe the system at any point in time. It is the set of values of all state variables in the system, the state variables being the attributes of all the entities (or objects of interest) in the system. Customer arrivals or departures (service completion) can change the system state.

3. You have been assigned the task of simulating a complex system and you have collected data about an input process to the system. You have constructed an empirical probability density function with the data listed below. How would you create values to represent this input process in your simulation?

Value	Cumulative probability	
-3.663	.1	
-0.71	.2	
0.208	.3	
0.807	.4	
1.203	.5	
1.679	.6	
2.119	.7	
2.551	.8	
2.988	.9	
3.618	1.0	

[3B2]

Answer 1: Generate a uniformly distributed random number between 0 and 1. Use that number to lookup values in the c.d.f. column. Use the corresponding number in the value column as a value for the R.V.

Answer 2: The distribution resembles a normal distribution. Find a mean value and variance that approximates the empirical data and use a normal R.V. generator to generate the random inputs.

I would hope the students use the graph area above to try plotting the c.d.f. to see that it is gaussian-like.

4. A factory assembly line produces subassemblies at an average rate of 100 units per hour with a distribution in completions/hour that can be modeled as a Poisson process. The assemblies are sent to one of five testing stations. Each testing station randomly receives 20% of the assemblies. The testing stations (staffed by Able, Baker, Charise, Doris, and Pat) and are capable of testing 30, 25, 15, 22, and 40 units per hour, respectively. What are the average numbers of assemblies waiting for each testing station?

[1A3]

The poisson arrival process can be split into five processes, based on the random splitting property. Each testing station sees an average arrival rate of 20 units per hour. Each station, except Charise's, has a capacity that exceeds the arrival rate, so the number of units waiting at Charise's station grows without bound. For the other stations, the average number of assemblies waiting can be found from Little's equation: the average arrival rate times the average service time per item. This is 20 per hour times  $1/30$ ,  $1/25$ ,  $1/22$  and  $1/40$ , so the average number of units waiting at the stations are .667, .8, .9091, and .5.

5. For the assembly line of problem 5, find the average utilization of Able's testing station in terms of the average utilization of Pat's station.

[1A2] .8

[1A3] .2

By the property of networks of queues load splitting across multiple servers, the server utilization is inversely proportional to the server capacity. Since Pat can test 40 units per hour, compared to Able's ability to test 30, Able's station utilization will be 40/30ths of Pat's.

$$r_{Able} = 4 * r_{Pat} / 3$$

6. A student commutes from Newark to Hoboken and can take the PATH or NJ Transit train. They attempt to arrive in time to take the train, but due to the random delays they encounter, they are in time to take the train 90% of the time. What is the probability that of the 20 days they commute in February, they will take the PATH no more than 5 times?

[1A1] .2

[1A3] .8

Consider this as a Bernoulli trial with a sample size of 20 and  $p=.1$ .

$$P(X \leq 5) = \sum_{x=0}^5 \binom{20}{x} (.1)^x (.9)^{20-x}$$

where

$$\binom{20}{x} = \frac{20!}{x!(20-x)!}$$

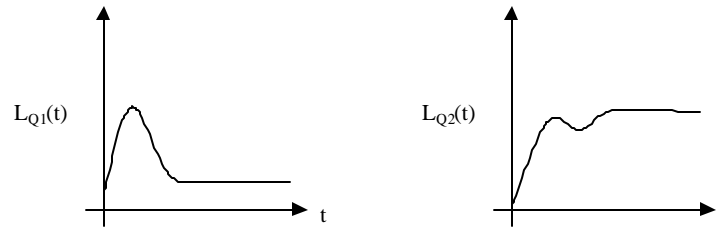
$$= (.9)^{20}/0! + (20)*(.1)*(.9)^{19}/1! + (20)(19)*(.1)^2*(.9)^{18}/2! + (20)(19)(18)*(.1)^3*(.9)^{17}/3! + (20)(19)(18)(17)*(.1)^4*(.9)^{16}/4! + (20)(19)(18)(17)(16)*(.1)^5*(.9)^{15}/5! = .989$$

7. What advantage does the SPT queuing discipline offer over FIFO or LIFO? How might this influence queue statistics?

[1A3]

SPT (Shortest Processing First) serves the customers with the shortest processing time first. A large number of short jobs can be completed, rather than having a large number of customers stuck behind a single long service time customer. Neither FIFO or LIFO offer this advantage – their service order is fixed by the order customers arrive. SPT will reduce the waiting time of the customers with shorter than average service times, in general reducing the average waiting times and reducing the average queue length.

8. You observe the behavior graphed below in a two different queuing systems. Both systems have the same average customer arrival rate,  $I$ , but have different mean service times  $m_1$  and  $m_2$ . Assuming that the scale of the graphs of  $L_Q(t)$  are the same, what observation can you make about  $r_1$  versus  $r_2$ ?



[4A3]

The queue length of System 1 and 2 appears to have reached steady state values, but the long term average queue length  $L_Q$  appears to be greater for System 2. This suggests that the server utilization,  $r$ , is probably higher in System 2 (which means that the mean service time  $m$  is probably longer for System 2).

9. You have been asked to design the exit road for the Podunk, New Jersey motor vehicle inspection station. There are entrances into the station from Highways 45 and 49. Traffic on these two roads can be modeled as Poisson processes. On any given day, the average traffic rates are 300 and 500 cars per hour, respectively. On average, 2% of the cars on each road will enter the inspection station. The station is capable of processing 20 cars per hour. What is the expected average arrival rate at the exit road of the inspection station?

[1A3]

The arrival rates from Highways 45 and 49 into the inspection station are 6 and 10 cars per hour (2% of 300 and 500), respectively. Using the random pooling property, this total arrival rate at the inspection station is 16 cars per hour. Since the inspection station is capable of processing cars at 20 cars per hour, the station queue is bounded, so the average departure rate is equal to the average arrival rate or 16 cars per hour.

10. Able and Baker bought their own donut shop where customer arrivals can be described by a Poisson process with an average arrival rate of 6 customers per hour. What is the probability that the first customer arrival will occur in the first 5 minutes the store is open?

[1A3] .9

[4A2] .1

For a Poisson arrival process, the probability that the first arrival will occur in the interval  $[0, t]$  is  $p = P(A_1 \leq t) = 1 - e^{-It}$ .  $I = 6$ ,  $t = 5/60$ .  $p = 1 - e^{-.5} = .3935$

**Do any 2 of the following 4 questions. Do more than 2 for extra credit.**

11. At  $t=15.5$ , the Future Event List for a system simulation you have written is:  $\{(1,15.5), (1,17.6), (2,18.3), (1,21.5), (2,25.5), (2,25.1), (2,26.4), (1,30.2)\}$ , where a Type 1 event is a customer arrival and a Type 2 event is service completion by the single server in the system. What observations can you make about the FEL?

- (1) the FEL is not sorted in order of time (the event at 25.1 is after the 25.8 event)
- (2) there are more departures than there are arrivals – the (2,26.4) event has no corresponding arrival [this might be an ambiguous question – you could assume that there was a customer in the system prior to  $t=15.5$  whose service time doesn't complete until 26.4. No points off if you don't notice this possible issue]

12. Hard Knocks University has 200 workstations sharing 4 printers in the computer room for the use of the college's 1800 undergraduate and 3000 graduate students. Print jobs arrivals can be modeled as a truncated normal distribution during the day having a mean arrival time of noon and a standard deviation of 3 hours. The page count is uniformly distributed between 1 and 30 pages. Print jobs are sent to a dual processor print server computer, where they are spooled on one of two hard disks while waiting for the first printer to be available. In the standard notation of queuing systems, how would this queuing system be characterized? (e.g., as an M/M/1 queue? Or some other type?)

G/G/4/N/3800: General arrival, general service (assuming service time ~ length), 4 servers, a limit of N jobs in the queue (it has to be finite, based on the hard disks) and 3800 students in the population (again, finite). The hard disks is not servers, nor are the workstations or the print server machines, the printers are the servers, so  $c=4$ . (Note: You might also consider this to be a G/G/4/N/200 if you consider jobs to be generated by workstations, but this assumption must be stated explicitly. For instance, workstations could generate print jobs that correspond to task completions that are not directly controlled by users.)

13. Why is a linked-list often used as the data structure for the FEL?

It is convenient to insert events into the linked list at any arbitrary point without moving data around. Also, the head of the list (the first element) can represent the imminent event, making it easy to process and remove again, without reordering other items in the list. Finally, the linked-list easily maintains data structures whose length changes during the simulation.

14. We are using a linked list for the FEL in a simulation. What is the time of the next scheduled service completion/customer departure? Head of list = 2.

Array position	Record contents	Next item
1	(arrival, 3.0)	3
2	(arrival, 5.1)	7
3	(departure, 5.2)	11
4	(departure, 8.0)	5
5	(departure, 8.5)	5
6	(departure, 6.0)	2
7	(arrival, 5.9)	9
8	(arrival, 7.0)	4
9	(departure, 6.5)	8
10	(arrival, 1.0)	4
11	(departure, 1.5)	11

6.5

#### ABET criteria

- 1A1 - Recognize mathematical parameters as if they were physical variables and vice-versa
- 1A2 - Be able to follow general mathematical concepts of derivation of engineering or scientific result and possess the mathematical skills to link those concepts
- 1A3 - Be able to understand the relevance of the mathematical results to physical applications
- ~~•1A4 - Be able to articulate algorithmic thinking through flow charts~~
- ~~•2B1 - Use software for preparing, transmitting, and displaying multimedia documents~~
- 3B2 - Have the ability to use computational tools for finding graphical, numerical, statistical, and analytic solutions to problems
- 3B3 - Have the ability to use systems simulations appropriate to engineering practice
- 4A2 - Be able to identify input, output, and operating variables as appropriate in various units
- 4A3 - Be able to identify technical relationships between the input, output and variables and use the relationships to predict mutual changes.