E245B
HW #6, 10/25/01

Problem 5.64

For the network in Figure P5.64 choose C such that

$$v_o = -10\int v_s dt$$

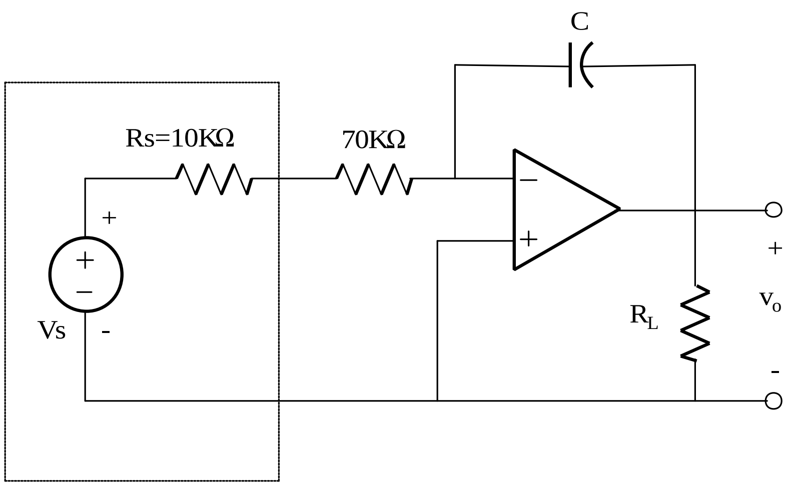
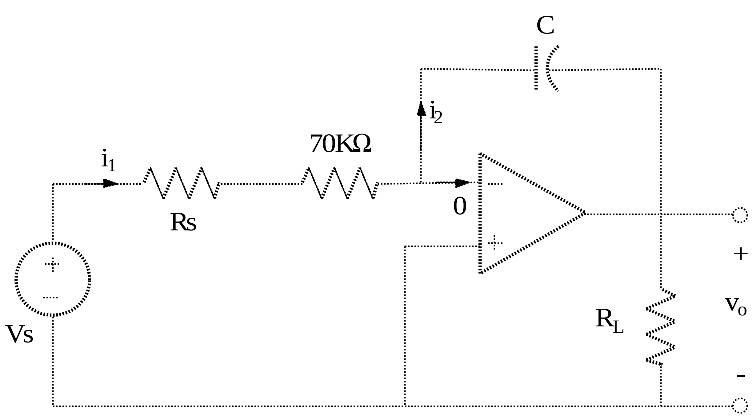


Figure P5.64

Suggested Solution



$Rs = 10K\Omega$

$v_o = -10\int v_s dt$

$\mathrm{R}_{eq} = Rs + 70K = 80K$

Using ideal op-amp assumptions,

$i_1 = i_2$

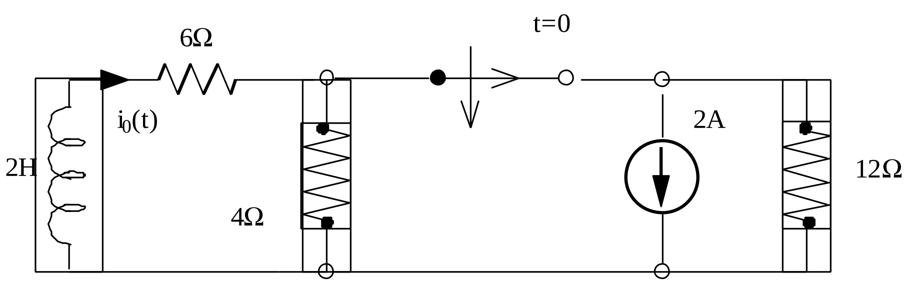
$$\frac{v_s - 0}{\mathrm{R}_{eq}} = -C\frac{dv_o}{dt} \Rightarrow v_o = -\frac{1}{\mathrm{R}_{eq}C}\int v_s dt$$

So,

$$\mathrm{R}_{eq}C = \frac{1}{10} \Rightarrow \boxed{C = 1.25mF}$$

Problem 6.5

In the network in Fig P6.5, find $i_0(t)$ for t>0 using the differential equation approach.



Suggested Solution

$$i_L(0^-) = 2\left(\frac{3}{3+6}\right) = \frac{2}{3}\ \mathrm{A}$$ using current division

$$L\frac{di_o(t)}{dt} + 10i_o(t) = 0 \qquad 10 = 6 + 4$$

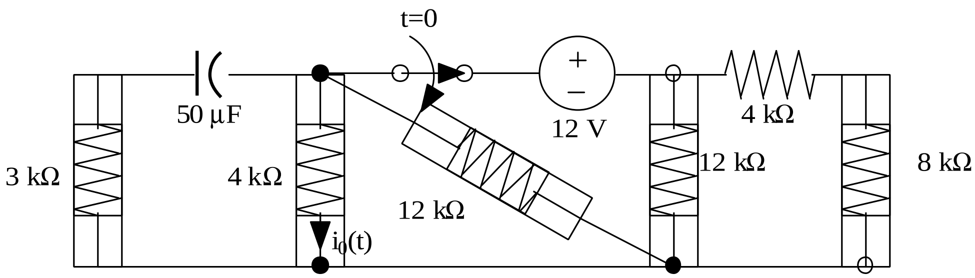
$$\therefore\ i_o(t) = K_2 e^{-5t}\ \mathrm{A} \qquad 5 = \frac{10}{L} = \frac{10}{2}$$

So, $i_o(0) = \frac{2}{3} = K_2$

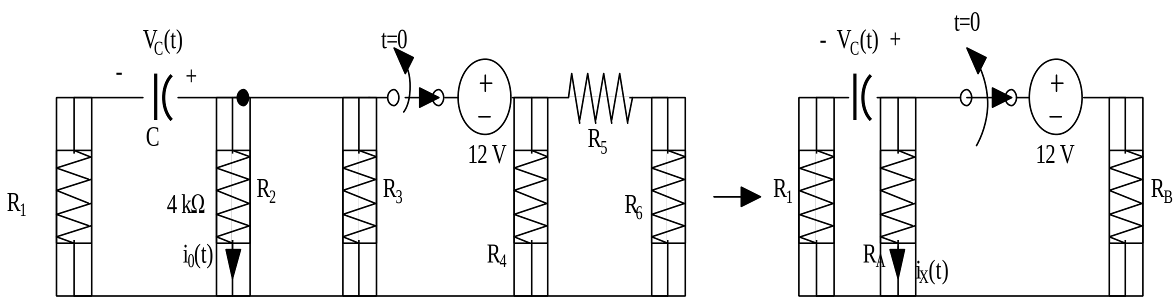
Then, $i_o(t) = \frac{2}{3} e^{-5t}$ A, $t > 0$

Problem 6.21

Use the differential equation approach to find $i_0(t)$ for t > 0 in the circuit in Fig. P.21 and plot the response including the time interval just prior opening the switch.



Suggested Solution



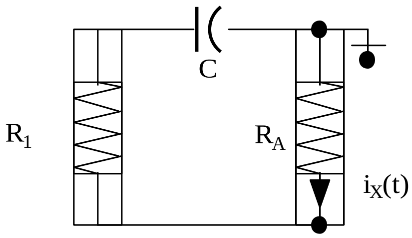
$R_1 = 3\ \text{k}\Omega \quad R_2 = R_5 = 4\ \text{k}\Omega \quad R_3 = R_4 = 12\ \text{k}\Omega \quad R_6 = 8\ \text{k}\Omega \quad C = 0.05 mF \quad R_A = 3k\Omega \quad R_B = 6k\Omega$

<u>For t < 0:</u> $\quad V_C(t) = \frac{-12R_A}{(R_A + R_B)} = -4V = V_C(0^-) = V_C(0^+)$

$$i_X(0^-) = \frac{V_C(0\text{-})}{R_A} = \frac{-4}{3}\ mA \quad i_0(0-) = \frac{i_X(0-)R_3}{(R_2 + R_3)} = -1\ mA$$

<u>*For t > 0:*</u> $\quad$ <u>By KCL:</u> $\quad \frac{C\, dV_C(t)}{dt} + \frac{V_C(t)}{R_1 + R_A} = 0 \Rightarrow \frac{dV_C}{dt} + \frac{10}{3} = 0$

For t > 0



$If\ \ V_C(t) = K_1 + K_2 e^{-\frac{t}{t}}, then\ \ t = 0.3\ S$ and $K_1 = 0$

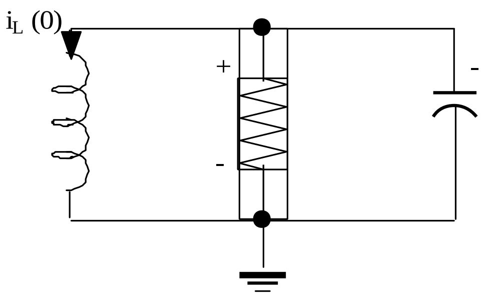
$V_C(0^+) = K_2 = -4V$ so, $V_C(t) = -4e^{-\frac{10t}{3}}V$

$But\ i_X(t) = \frac{-V_C(t)}{R_1 + R_A}$ and $i_0(t) = \frac{i_X(t)R_3}{R_2 + R_3} \Rightarrow$

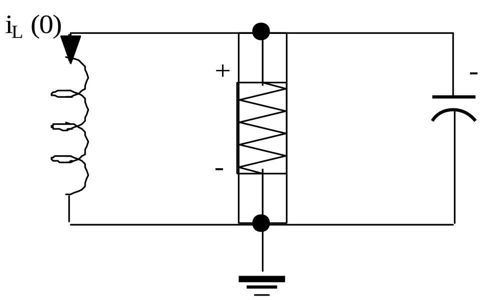
$$i_0(t) = 0.5e^{-\frac{10t}{t}}\ mA,\ t > 0$$
$$= 1\ mA,\ t < 0$$

Problem 6.61

For the underdamped circuit shown in Fig. 6.61 determine the voltage v(t) if the initial conditions on the storage elements are $i_L(0)$=1 A and v(0) = 10 V.



Suggested Solution



Find V(t) if i_L(0)=1A and v_c(0)=10v

$$\frac{d^2v(t)}{dt^2}+\frac{1}{RC}\frac{dv(t)}{dt}+\frac{1}{LC}v(t)=0$$

The characteristic equation is

$$s^2+8s+20=0$$

$$s=-4\pm 2j$$

so

$$v(t)=k_1e^{-4t}\cos 2t+k_2e^{-4t}\sin 2t$$

at

$$t=0, v_c=10$$

$$v(t)=k_1=10$$

then

$$\frac{dv(t)}{dt}=-2k_1e^{-4t}\sin 2t-4k_1e^{-4t}\cos 2t+2k_2e^{-4t}\cos 2t-4k_2e^{-4t}\sin 2t$$

at

$$t=0$$

$$\frac{dv(t)}{dt}=-4k_1+2k_2=-40+2k_2$$

also

$$\frac{cdv(t)}{dt}+\frac{v(t)}{R}+i_L(t)=0$$

at

$$t=0$$

$$\frac{dv(t)}{dt}=\frac{1}{c}(\frac{-v(t)}{R}-i_L(t))=\frac{1}{c}(\frac{-10}{R}-1)$$

if

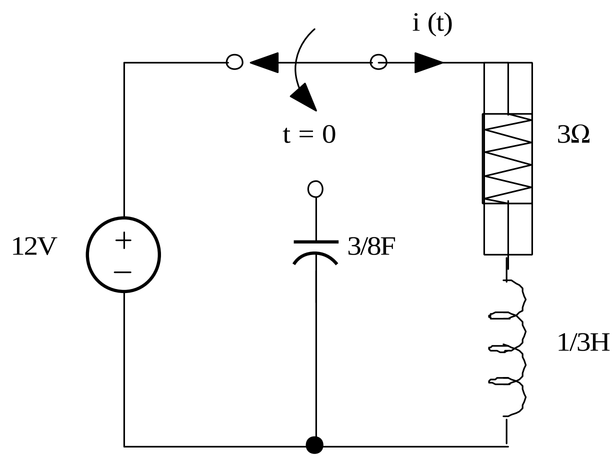
$$-40+2k_2=-120$$

finally

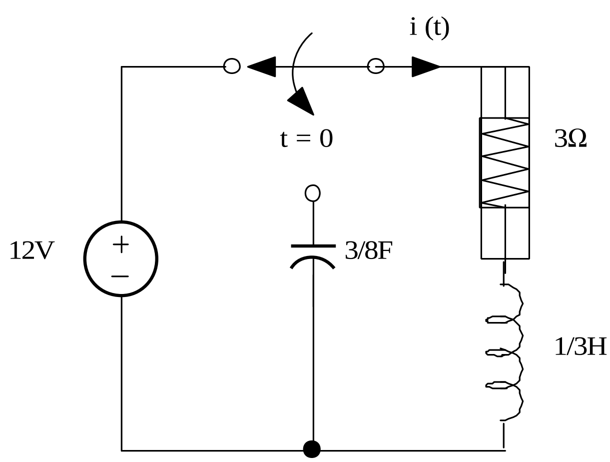
$$v(t)=10e^{-4t}\cos 2t-40e^{-4t}\sin 2t$$

Problem 6.67

Given the circuit in Fig. 6.67, find the equation for i(t), t > 0.



Suggested Solution



for

$t < 0$

$i_L = 12/3 = 4A, v_c = 0v$

for

$t = 0^+$

$i_L = 4A, v_c = 0v$

for

$t > 0$

Series RLC circuit

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 9s + s = 0 \Rightarrow (s+8)(s+9) = 0$$

since roots are real and unequal, overdamped and

$i(t) = k_1 e^{-t} + k_2 e^{-8t}$

$i(0) = 4 = k_1 + k_2$

$KCL:$

$$v_c(0^+) = Ri(0^+) + L\frac{di(t)}{dt}|_{t=0} \Rightarrow 0 = 12 + \frac{1}{3}(-k_1 - 3k_2)$$

so

$k_1 + k_2 = 4$

$k_1 + 8k_2 = 36$

$$k_1 = \frac{-4}{7}, k_2 = \frac{32}{7}$$

$$i(t) = \frac{32}{7}e^{-8t} - \frac{4}{7}e^{-t}A \quad \text{for t>0}$$