Signal to Interference Prediction
with Arbitrary Number of Co-Channel Interferers

Abstract:

I. Introduction

Consider a network of nodes, which employ adaptive communication links, where data is transmitted opportunistically when the channel is in a favorable state. Due to a flexible spectrum sharing policy, besides other nodes of the same system coordinating by some MAC layer protocol, assume there are other co-channel interferers which do not cooperate. They transmit independently and continuously at full power. One such possibility could be microwave links.

Let the receiver measure the interference and feed back the signal to interference ratio (SIR) to the transmitter as a measure of the achievable rate. It is assumed that the receiver can obtain a coherent channel estimate blindly or through knowledge of the pilot sequences of each of the interferers.

Either the transmitter or receiver, or objects in the environment could be moving, so all signals will fade in time. Since it takes a finite amount of time to relay back the SIR, it will be stale to degree the channel is varying. By using prediction we can improve the SIR estimate when channels are fading quickly. Both signal and interference powers are modeled as auto-regressive (AR) stochastic processes. We derive the probability distribution function of the prediction error to determine the backoff margin required for a percentage of packets to be decoded successfully. The back off is a function of the predicted signals and the error, as well as those interferers which are not predicted. Since the backoff reduces the rate to improve delay, the trade off between delay and rate may be determined for different scenarios. This issue related to the quality of service (QOS).

Downlink wireless packet systems use channel knowledge to adapt transmission rate on packet by packet basis as the channel fluctuates. A key part of the downlink access scheme is the scheduler, which uses this knowledge to transmit to mobiles when their channel capacity is high, and defers when it is not. The channel is estimated at the mobile from downlink pilots and fed back regularly on the uplink. By the time the information is ready to be utilized at the base station, several milliseconds may have passed since the measurement was taken. This delay is the sum of the measurement interval, uplink MAC, transmission, and internal processing delays. Although we are considering a frequency duplexed system, it may be noted that even in a time duplexed system, there will be a delay between when the uplink pilots are transmitted, and when downlink transmission begins. Depending on the speed of the mobile, the delay, and the channel's spatial/temporal properties any information will be out-of-date or stale to varying degree. Two issues arise: first the scheduler makes 'wrong' decisions, and second the transmission rate must be backed off to ensure that this rate does not exceed the actual channel capacity. The effect of this delay was studied in [1] by Avidor et. al.

Channel prediction has been proposed, to support mobile velocities [2-4]. If a mobile could feed back a predicted version of the channel, the gains by the described feedback system could be maintained. For example, in CDMA 2000 EV-DO there is 2.5 slot, or 4.1 ms delay from the measurement to the downlink slot. Using the rule of thumb $f_d=1/(20*t_{delay})$, prediction may be beneficial when the mobile velocity exceeds 4.2 mph.

At high mobile velocities, hybrid automatic repeat request (HARQ) and incremental redundancy (IR) allow packets to finish early if a portion is received successfully, thus gaining back what was lost due to increased fade margins. Yet this gain is only partial, as [5] compares performance at 2 mph to 13 mph showing a loss of 67% without HARQ but only 33% loss with HARQ/IR. It may be noted that with on-time channel feedback or prediction, gains are not just maintained, but system throughput *increases* with mobile velocity. Besides adding to the delay, another reason not to simply depend on HARQ/IR is that this process is expensive in terms of power, that is it takes multiple tries to decode a single packet.

Admittedly the channel prediction itself will have errors, but there must be a point when the error is small enough that prediction and margin will improve throughputs. When the prediction error and margin become large, other techniques such as IR and TX diversity should be employed. The mean prediction error may be easily estimated, since the prediction and the actual value of the channel will both be available, after a short delay. We desire a backoff margin such that most of the packets can be decoded successfully on the first attempt. This requires an estimate of the true density of the SIR, given the best estimate, which is our prediction. That is, given a prior predicted SIR, determine the posterior probability. An alternative approach taken in [12] is to model the effect of prediction error on uncoded BER.

Geometric Model for Interference.

Consider a hexagonal cellular layout as in Figure 1, showing the serving base marked by 1, and the two strongest interfering bases at 2 and 3. Each base station with a three sectored antenna, and the dotted lines mark the nominal beamwidth. On the downlink, neglecting shadowfading, average signal to interference at a point A is -.8 dB. As one is closer to the serving base the SIR is higher. Even though the SIR is very low, it is still possible to run an adaptive link, though of course at much lower rates as compared to locations with higher SIR. Prediction can alleviate the feedback delays, and ensure an accurate report. Since prediction requires good channel estimates, we note this is possible because on the pilot there is typically significant processing gain i.e. coherent averaging, which suppresses the interference. When two rings of interfering cells are considered, typically 40 % of the cell area, primarily around the outer boundary of the cell area, faces an signal to interference ratio of 0 dB or lower.

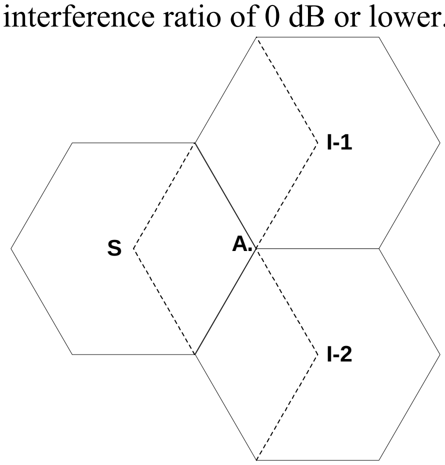

Figure 1. Three sectored cell geometry with two strongly interfering base stations.

PF Scheduling with fading interferers.

Consider the situation where the users are at the outer edge of the cell, and all have the similar fading statistics at the mobile at location A. The maximum benefit to be gained by from rate feedback in downlink scheduled systems has been studied under certain simplifying conditions. Assume that proportional fair algorithm is used for scheduling, and T_c, the throughput averaging decay constant, is infinite. Let the scheduler be in steady state, and the all mobiles have the same average SIR. Under these assumptions the asymptotic throughput of the PF system becomes the max rate scheduler. For arbitrarily signal and interference powers, each fading, the throughput is given by [1, eq. 17]. The throughput for signal only fading was given earlier in [4]. As an example, let the average SIR be 0 dB, and let there be two interferers at –3 dB, and other unfaded noise and interference at –10 dB. The rate R(t) is given by $\log_2(1+\text{SIR})$. Figure 2 shows the system throughput in bps/Hz vs. number of mobiles for the fading and non-fading case. Slightly more throughput when the interference fades as well as the signal is observed.

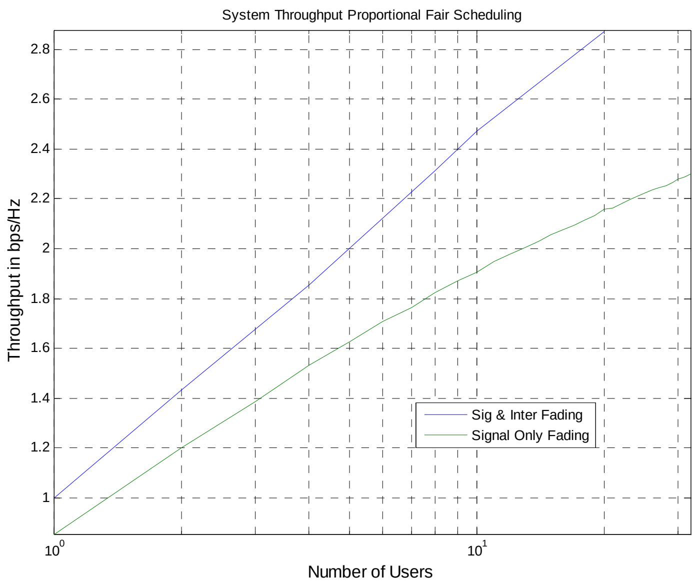

Figure 2: Asymptotic throughput for proportional fair vs. number of users for signal only fading, and signal and interference fading. Average SIR is 0 dB in both cases.

Determining the Backoff

Mathematically the objective is to determine a backoff factor θ such that

$$P_r\{\theta\hat{\gamma} > \gamma\} < \Omega \qquad (1)$$

where Ω is the outage probability, y is the true signal to interference ratio (SIR), and $\hat{y}$ is the predicted SIR. When y is less than the fed back $\hat{y}$ this is called an outage. It is assumed that actual transmission rate is some function of y, such as $\log_2(1+y)$. Rates are assumed to be continuously adjustable, although in practice a

The narrowband received signal and interference at the mobile may be given as

$$r(t) = h_s(t)D_s(t) + \sum_{k=1}^{M} h_k(t)D_k(t) + n(t) \text{ volts} \qquad (2)$$

where *h(t)* is the complex channel gain, *D(t)* is the modulated data symbol and are $i.i.d.\ N(0,1)$, and *n(t)* is thermal noise. *h(t)* is assumed to be complex Gaussian with time variation bandlimited to $F_d = v/\lambda$, where v is the mobile's own velocity. Interfering basestations continuously transmit at full power. This is justified by the fact that the multi-user wireless channel capacity is orders of magnitude less than a single wire, and is thus likely to be on all the time even with few users. Another approach taken in [12], is to postulate high capacity link so that the transmit power will be bursty just as the data is.

The predicted value of the complex envelope of the signal and interference power is obtain by linear prediction. Using an autoregressive model of the fading process, the Yule Walker equations may be solved to obtain the optimal coefficients []. Unbiased signal power prediction may be obtained by squaring the predicted envelope and subtracting the bias. Since the input distribution is complex Gaussian, and the output distribution will also be complex Gaussian after linear filtering. The square of the input and output will then have a bivariate chi-square distribution. Using the optimal coefficients the conditional signal power given the unbiased predicted power is [10, pg. 172, eq. 7.46],

$$f(p\,|\,\hat{p}) = \frac{1}{\sigma_e^2}\exp\left(-\left(\frac{p+(\hat{p}+bias)}{\sigma_e^2}\right)\right) I_0\left(\frac{2\sqrt{p(\hat{p}+bias)}}{\sigma_e^2}\right). \tag{3}$$

The bias is equal to $-\sigma_e^2$. The mean square channel error σ_e^2 is defined here as the difference between the predicted complex envelope and the true envelope. While, the mean square prediction error is defined as the expected value of the square of the difference of the square of the envelopes. Eq. (3) is the density of a bivariate chi-square or equivalently as a noncentral chi-square, and with a change of variable may be recognized as a Rice distribution. The conditional SINR may be defined as:

$$\gamma\,|\,\hat{\gamma} \equiv \frac{S\,|\,\hat{S}}{\sum_{k=1}^{M} I_k\,|\,\hat{I}_k + \sigma_n^2}. \tag{4}$$

where σ_n^2 is variance of additive white noise.

It is reasonable to assume that the mobile will have only be able to track and predict a limited number of interferers. Furthermore as interferer powers decrease the SIR of the channel estimate decreases, and the prediction accuracy is degraded. So let M_1 be the number of predicted interferers. Let the mean value of the remaining M-M_1 interferers be known. For each of the predicted interferers, $I\,|\,\hat{I}$ is scaled noncentral chi-square *rv*, while for the remainder $I\,|\,\hat{I}$ is scaled central chi-square *rv*. That is,

$$\begin{aligned} &I_k\,|\,\hat{I}_k \sim \sigma_{h,k}^2 \chi^2(2,\lambda_k) \\ &\lambda_k = \begin{cases} \sigma_{e,k}^2/\sigma_{h,k}^2 & 1<k \le M_1 \\ 0 & M_1<k<M \end{cases}. \end{aligned} \tag{5}$$

Since the powers are not all equal an exact expression is even quite complicated to evaluate numerically on a computer, for example see [5]. A well known approximation often used is due to Satterthwaite [6], which equates the first and second moments to a central chi-square *rv*. First, let

$$Z = \sum_{k=1}^{M} I_k \mid \hat{I}_k \qquad Z \,\square\, a\chi^2(f)\,. \tag{6}$$

The equating the first two moments of Z gives:

$$\begin{aligned} u_z &= af = \sum_{i=1}^{M} a_i(\lambda_i + f_i) \\ \sigma_z^2 &= 2fa^2 = 2\sum_{i=1}^{M} a_i^2(2\lambda_i + f_i) \end{aligned} \tag{7}$$

Solving for f and a yields

$$\begin{aligned} a &= \sigma_z^2 / 2u_z \\ f &= 2u_z^2 / \sigma_z^2 \end{aligned} \tag{8}$$

Another technique that approximates the noncentral chi-square [7] matches only the mean, and in general does not match the variance. Now the density of $\gamma \mid \hat{\gamma}$ is observed to be a singly noncentral F, a type of mixture distribution. Using the inverse CDF the backed off SIR value may be obtained,

$$\theta\hat{\gamma} = F_{\gamma|\hat{\gamma}}^{-1}(\Omega)\,. \tag{9}$$

To compute the pdf of $\gamma \mid \hat{\gamma}$ first order saddlepoint approximation [9] was used with the roots obtained in [10].

As an example of $\gamma \mid \hat{\gamma}$, take the value of interference powers, predicted values, and errors given in Table 1.

	Avg. Power	Normalized MSE	Predicted Power
Signal	0 dB	.02	2
Interferer 1	-3 dB	.02	.5
Interferer 2	-6 dB	NA	NA
Interferer 3	-8 dB	NA	NA
Interferer 4	-8 dB	NA	NA

Table 1. Interferer powers, and predicted powers.

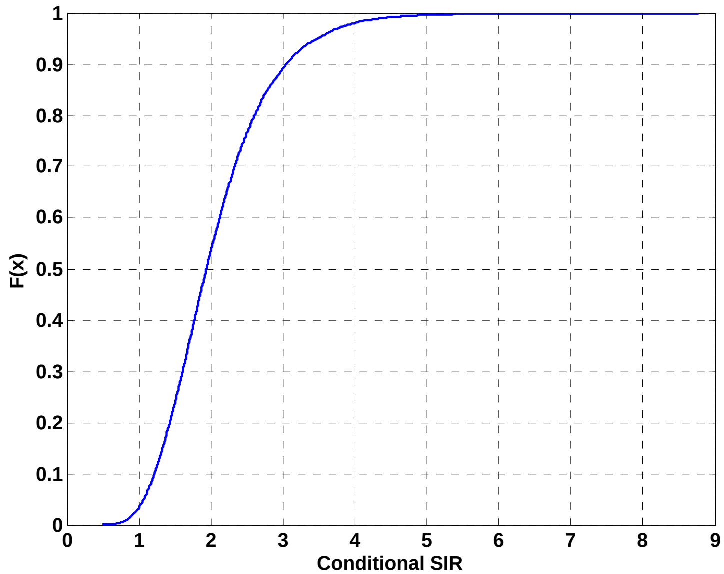

Figure 2. Conditional SIR for the powers given in Table 1.

The predicted SIR is 2/(.5+.25+.16+.16) = 1.9. Allowing for a backoff of .64 (-1.9dB) sets the outage probability to 10%.

Proportional Fair Scheduling with Prediction

Let each mobile measure the downlink pilots of nearby basestations and predict the value of the SIR during the downlink timeslot. The backoff factor is determined locally to maintain particular outage. The base station receives all the backed off SIRs and runs the proportional fair scheduler. Specifically, the mobile for the *t*-th time slot is chosen by

$$J(t) = \arg \max_{1<i<N_{mobile}} \left(\frac{\theta \hat{\gamma}_i(t)}{T_i(t)} \right) \qquad (10)$$

where

$$\begin{aligned} T_i(t+1) &= T_i(t) * (1 - 1/T_c), \quad i \neq J(t) \\ T_i(t+1) &= T_i(t) * (1 - 1/T_c) + \mathrm{R}(\theta \hat{\gamma}_i(t))(1/T_c), \quad i = J(t) \end{aligned} \qquad (11)$$

Consider a system without prediction, called here most recent value (MRV). The mobile measures the SIR and the backoff transmission to allow low outage is somehow given. Consider another system that does no feedback or channel state scheduling, but has two transmit antennas use for diversity. Mobiles are served round robin. Interference is assumed to be unfaded. A margin is again required to obtain low outage, but is now based on the signal fading, which is reduced due to the TX diversity. Note, let the distribution of the fades with TX diversity be the same as for switched RX diversity.

N=10. The outage is set to 10% for all systems. The time slot length is 1.7ms. In the predicted system all mobiles are assumed to have the same velocity. They each predict the value of the signal and two interferers. The average SIR of all the systems is 0 dB. The autoregressive predictor utilizes the theoretical autocorrelation of the Jake fading process rather than estimating it. The filter order is 8, and the channel estimate SNR is 20 dB. Figure 3 shows the throughput of the three systems as a function of mobile velocity.

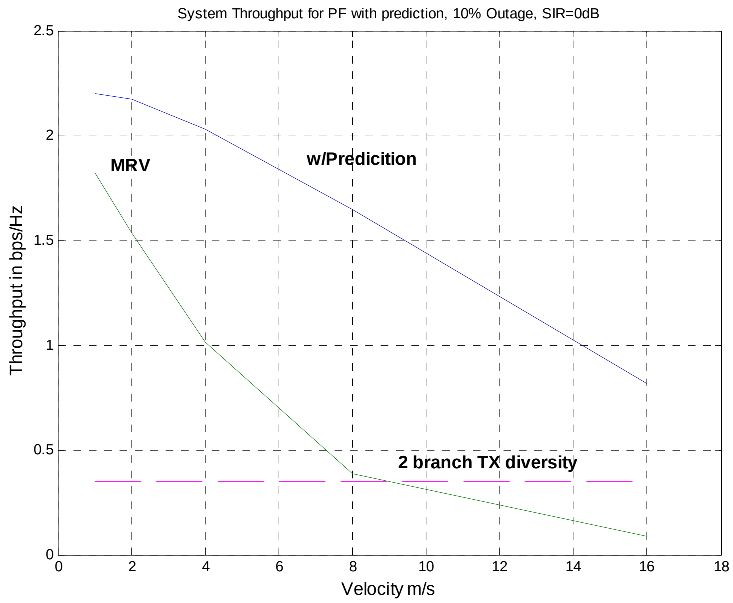

Figure 3. Comparison of the cell throughput of three downlink packet systems.

From the figure we observe that both predicted and MRV system lose system throughput as a linear function of velocity. MRV throughput drops immediately, while the predicted system begins drops at a lower rate at 2 m/s or 4 mph. As prediction interval in wavelengths increases the predicted system throughput drops due to the increase in prediction error and increase in backoff needed to maintain 10% outage. The predicted system is seen to extend the usefulness of adaptive transmission and scheduling to higher velocities. Channel prediction doubles the velocities at which the crossover point in throughput between the feedback system and the diversity system occurs.

Conclusions

The performance of a downlink packet system with channel prediction at the mobile has been evaluated. An FDD system was assumed, so that the downlink pilots are measured, and the SIR is feed back to base on the uplink. Thus without channel prediction channel state information is stale to the degree at which the channel varies. In the work we have modeled the situation where both signal and interference are fading and can be tracked and predicted. The sum of the conditional interference powers was modeled as a scaled chi-square variable, and the conditional SINR was shown to have a singly noncentral F distribution. The predicted system was seen to extend the usefulness of adaptive transmission and scheduling to higher mobile velocities. Channel prediction can double the velocities at which the throughput crossover point between the feedback system and the diversity system.

References:

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