EVALUATING PERFORMANCE OF VARIOUS LOCALIZATION ALGORITHMS IN WIRELESS AND SENSOR NETWORKS

Jasmin Desai
jdesai1@stevens.edu
Dept. Elec. and Comp. Engineering
Stevens Institute of Technology,
Hoboken NJ 07030

Uf Tureli
utureli@stevens.edu
Dept. Elec. and Comp. Engineering
Stevens Institute of Technology,
Hoboken NJ 07030

ABSTRACT

In this paper, recent localization algorithms are analyzed under a common one hop network scenario. The performance of localization is affected by physical parameters in a real wireless environment such as anchor node location and quantity, and error in the measured distance. The numerical analysis presented in this paper can be used to choose among localization algorithms to satisfy practical constraints such as number of anchors, nodes, geometry of anchors and computational efficiency.

I. INTRODUCTION

Localization performs a key role in higher level services in current wireless sensor, ad-hoc and cellular networks. Sensor network comprises small size, low power and low cost nodes with various sensors, e.g. temperature, audio, humidity, proximity and others. Typical applications are in surveillance, security applications and monitoring. Location awareness in such networks is crucial due to the position dependent data collection and to reduce overhead for routing data. GPS is not an appropriate solution for localization due to cost, power consumption and its limited application to outdoor environment [1].

Popularity of wireless LAN and Bluetooth networks has created security issues and challenges for various location based attacks. Monitoring and tracking the location of wireless nodes reduce these security threats with providing location of attacker [2]. Furthermore, knowledge of location is greatly helpful in routing algorithms and in location critical applications, e.g. first responders and emergency calls.

The essential part of localization is range measurement and can be performed by time of arrival (TOA), time difference of arrival (TDOA) or received signal strength indication (RSSI) measurements [3]. Since many devices already have the ability to measure RSSI, we consider ranging using RSSI measurements. A simple radio frequency propagation model is assumed for range measurement and all algorithms are analyzed under this model. We have implemented various algorithms based on multilateration rather than triangulation. These selected algorithms are maximum likelihood estimation (MLE) [4, 5], modified multidimensional scaling (MMDS) [6], Malguki spring model (MSM) [7] and weighted multidimensional scaling (WMDS) [8].

The rest of the paper is organized as follows. In section II the system model is proposed. Section III provides a brief summary of all algorithms to be analyzed. Section IV covers simulation results and discussion regarding the performance of each algorithm. Comparison table is presented in section V. Finally, we conclude with future work in section VI.

II. SYSTEM MODEL

A. *Propagation Model*

In this paper, the performance of localization algorithms is evaluated under a common scenario. Instead of a very complicated indoor wireless channel model, a simple path loss model is assumed. The signal power in an actual environment decays with the distance [9]. The loss of power depends on various obstacles between transmitter and receiver. The overall effect results in lognormal distribution of received power at receiver. The mean received power by receiver at distance d is expressed in dB by,

$$\bar{P}_r(d) = \bar{P}(d_o) - 10n\log_{10}\left(\frac{d}{d_o}\right), \tag{1}$$

where $\bar{P}(d_o)$ is mean power received at reference distance d_o. For most indoor applications d_o=1 meter and the power at that distance is calculated by free space path loss formula. The path loss exponent n is determined by environmental variables and surrounding structure [9]. The received power is Gaussian distributed with mean power expressed by equation (1)

$$P_r(d) = \bar{P}(d_o) - 10n\log_{10}\left(\frac{d}{d_o}\right) + \varepsilon_{dB}, \tag{2}$$

where ε_{dB} is Gaussian distributed random variable with zero mean and variance σ^2, $N(0, \sigma^2)$. Based on this model, the distance estimation is performed by,

$$\tilde{d} = d_o 10^{\frac{-(P_r(d) - \bar{P}(d_o))}{10n}}. \tag{3}$$

If variance is zero and path loss exponent n is known by analytical or empirical study then distance measurement is perfect. However, that is not the case in practical environment where variance varies up to 2 dB, which corresponds to error in distance.

B. *Network Setup*

The performance of each algorithm is verified under the specified single hop network scenario, where the radio range is sufficient to be fully connected. Anchor nodes are defined to be those who know their position in advance by using GPS or having fixed geographic location with coordinate assignment. Localization algorithms are used to estimate unknown node positions. Figure 1 shows network setup with 20 unknown nodes

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and 7 anchor nodes. These anchor nodes position is arranged in several ways to examine anchor position dependency of algorithms. Unknown nodes are uniformly distributed in the area of 16×16 meter^2 large room.

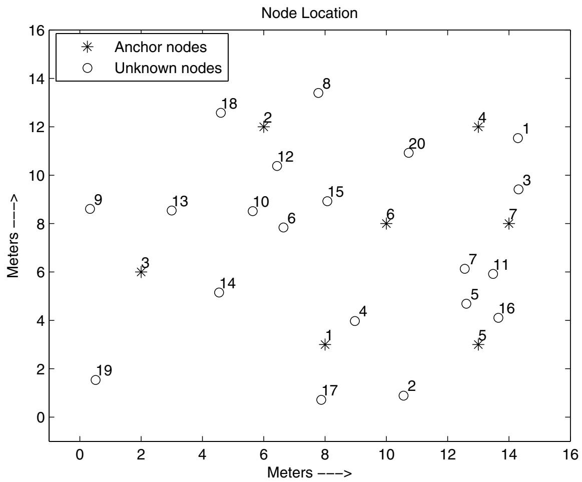

Figure 1: Network Setup

In Figure 1, all nodes are within communication range of each other, where full network connectivity is achieved without any communication range restrictions [10].

III. ALGORITHMS

We have implemented five algorithms which represent the state of the art in wireless and sensor network positioning. In the following, a brief description for each algorithm is presented. Consider the network model with n anchor nodes and one unknown node for simplicity as shown in Figure 2. Here, the unknown node coordinate is (x_o, y_o) and anchor nodes are (x_i, y_i), $i = 1, 2, 3, ..., n$. Measured distance between anchor and unknown node is denoted by r_i, $i = 1, 2, 3, ..., n$ and between anchor nodes is denoted by r_{ij}, $i = 1, 2, 3, ..., n$;$j = 1, 2, 3, ..., n$.

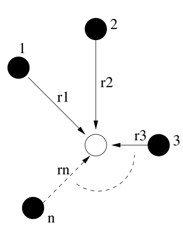

Figure 2: Simplified Localization Model

A. *Maximum Likelihood Estimation (MLE)*

This algorithm estimates the position of unknown nodes by minimizing difference between measured and estimated distance. This estimation can be performed using minimum mean square error (MMSE) criterion. Our study for localization is limited to two dimensions only. In this case, MMSE needs more than 3 anchor nodes to resolve unknown node location accurately. We have defined error e_i between measured and actual distance by,

$$e_i = r_i - \sqrt{(x_i - x_o)^2 + (y_i - y_o)^2}. \quad (4)$$

Considering ideal case as this error becomes zero and rearranging terms again we will have

$$-x_i^2 - y_i^2 + r_i^2 = (x_o^2 + y_o^2) - 2x_i x_o - 2y_i y_o, \quad (5)$$

where $1 < i < n$.
Eliminate the squared terms of unknown nodes by using similar n^{th} anchor node equation,

$$-x_i^2 - y_i^2 + r_i^2 + x_n^2 + y_n^2 - r_n^2 = 2x_o(x_n - x_i) + 2y_o(y_n - y_i). \quad (6)$$

Using above equation for all anchor nodes we will have matrix which has well known solution of,

$$y = Xb, \quad (7)$$

$$b = (X^T X)^{-1} X^T y, \quad (8)$$

$$X = 2 \begin{bmatrix} (x_n - x_1) & (y_n - y_1) \\ (x_n - x_2) & (y_n - y_2) \\ \vdots & \vdots \\ (x_n - x_{n-1}) & (y_n - y_{n-1}) \end{bmatrix}, b = \begin{bmatrix} x_o \\ y_o \end{bmatrix}, \quad (9)$$

$$y = \begin{bmatrix} -x_1^2 - y_1^2 + r_1^2 + x_n^2 + y_n^2 - r_n^2 \\ -x_2^2 - y_2^2 + r_2^2 + x_n^2 + y_n^2 - r_n^2 \\ \vdots \\ -x_{n-1}^2 - y_{n-1}^2 + r_{n-1}^2 + x_n^2 + y_n^2 - r_n^2 \end{bmatrix}, \quad (10)$$

where b gives unknown node position estimate (x_o, y_o) with MMSE estimation.

B. *Modified Multidimensional Scaling (MMDS)*

MMDS is a variant of classical MDS which is independent of unknown node's estimated position for final translation, rotation and reflection. Therefore, MMDS performs better than classical MDS but does not resolve all the unknown nodes position in a single step. It only considers one unknown node at a time and estimates its location. In classical MDS, the dissimilarity matrix is defined by,

$$D = \begin{bmatrix} 0 & r_1^2 & r_2^2 & \dots & r_n^2 \\ r_1^2 & 0 & r_{12}^2 & \dots & r_{1n}^2 \\ r_2^2 & r_{21}^2 & 0 & \dots & r_{2n}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_n^2 & r_{n1}^2 & r_{n2}^2 & \dots & 0 \end{bmatrix}. \quad (11)$$

Double centering is performed on matrix D and eigen value decomposition is applied as,

$$B = -\frac{1}{2} J_{n+1} D J_{n+1}, \quad (12)$$

$$B = V \Delta V^T. \quad (13)$$

First, two eigen vectors are selected for two dimensional case and principal axis solution is derived by,

$$X^r = V_2 {\Delta_2}^{\frac{1}{2}}. \tag{14}$$

This solution only estimates the coordinates up to an arbitrary rotation and the requisite transformation matrix is derived by,

$$\Omega = \left({X^r}^T X X^T X^r \right)^{\frac{1}{2}} (X^T X^r)^{-1}. \tag{15}$$

Equation 15 depends on estimated position of unknown nodes and results in erroneous rotation. MMDS solves this problem and derives rotation matrix that only depend on anchor node position by modifying matrix B. The detailed mathematical derivations are found in [4].

C. Malguki Spring Model (MSM)

Malguki considers each distance as a spring and minimizes the cost function by overall elastic force of the network nodes. This is an iterative algorithm and has a tradeoff between system complexity and estimation accuracy. The adaptive gain should be carefully selected to improve the rate of convergence with reasonable mean square error. Some useful mathematical expressions are described below and can be found in detail in [7]. The cost function ϕ is defined by,

$$\phi(\vec{v}) = \sum_{i=1}^{n} |r_i - |\vec{v} - \vec{v}_i||, \tag{16}$$

where $\vec{v}$ is vector for unknown location to be estimated and $\vec{v_i}$ is vector for anchor node location. Each spring has different length so the force $\vec{m_i}$ applied to the unknown node depends on the distance between unknown and anchor node by,

$$\vec{m_i} = (r_i - |\vec{v} - \vec{v}_i|)\, \hat{v}_i. \tag{17}$$

The cost function whose minimum is to be found can be expressed by using sum of individual forces from anchors as,

$$\phi(\vec{v}) = \sum_{i=1}^{n} |\vec{m_i}| = \sum_{i=1}^{n} \phi_i(\vec{v}). \tag{18}$$

To reduce this cost function, iteration is performed and $\vec{m}$ vector is translated into a displacement using positive scalar parameter γ:

$$\Delta\vec{v}^k = \gamma\vec{m}, k^{th} iteration \tag{19}$$

$$\vec{v}^{k+1} = \vec{v}^k + \Delta\vec{v}^k, (k+1)^{th} iteration. \tag{20}$$

D. Weighted Multidimensional Scaling (WMDS)

This is another iterative algorithm and uses weighted version of MDS incorporating local communication constraints within the network. The key benefits of this algorithm are heavy weighting on more accurate range measurements and majorization method for iteration which guarantees to improve cost function. The cost function S is modified slightly for our purpose.

$$S = 2 \sum_{1\le i\le m} \sum_{i\le j\le m+n} w_{ij} \left(r_{ij} - d_{ij}\right)^2, \tag{21}$$

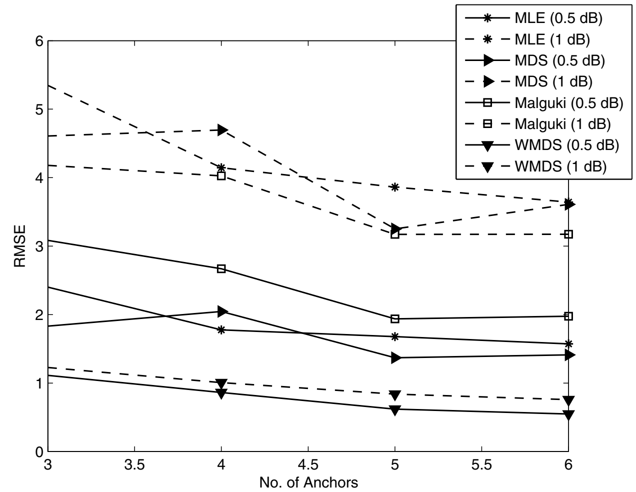

Figure 3: RMSE vs No of Anchor

where w_{ij} is proper weight, r_{ij} is measured distance and d_{ij} is actual distance between ith and jth nodes. The number of unknown and anchor nodes are defined as m and n respectively. We have eliminated multiple measurements in specified time interval to have a fair comparison with other algorithms. We have not considered any vibration or movement of anchor nodes here. The detailed derivation of this method is found in [8].

IV. Simulation Results

The performance of all algorithms is examined with numerical analysis using MATLAB. Performance metric is defined as root mean square error,

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left((x_o - x_{i,est})^2 + (y_o - y_{i,est})^2 \right)}, \tag{22}$$

where N is total number of runs and $x_{i,est}$ and $y_{i,est}$ is estimated position at each run. We have selected N=700 for all of the algorithms. RMSE dependence on number of anchor nodes and error in measured distance, while keeping other parameters constant, is examined. The communication range of each node is sufficient to reach all other nodes in the network assuming single hop network connectivity.

A. Effect of increase in anchor nodes

The network model is implemented as shown in Figure 1 and peak RMSE is limited by 6 meters. Anchor nodes are arranged in a way that their position dependency is easily verified. Anchor nodes are selected according to the node number assigned to them. The estimation of all 20 unknown nodes is averaged out to better rely on overall system performance rather than single node. Figure 3 shows results with 0.5 and 1 dB error in RSSI measurement for all algorithms. Malguki spring model has better performance with an increase in anchor nodes. The position error remains below 2 meters with 5 anchor nodes and

0.5 dB measurement error and increasing measurement error by 0.5 dB increases position error by 1 meter only. However, an important result which we note on the Malguki model is that the unknown node situated outside the polygon with vertices as the anchor nodes has a large position error. It is difficult for algorithm to find local minima for this case. Therefore, this algorithm is inappropriate in such scenario.

MLE also has performance improvement with more anchors. Position error remains below 2 meters with 4 anchors and 0.5 dB measurement error. Increase in measurement error by 0.5 dB increases position error by 2 meters, which is poor compared to Malguki. The performance curve for 1 dB error has great dependence on number of anchors because MLE performance improves with more measurements. However the performance improvement is not guaranteed as would be in the Malguki model. Performance deviates largely when some anchors are very close and others are far away. The measurement error with an anchor node far away from unknown node is very large compared to the error where the anchor node is nearby. Therefore, large position error occurs due to equal preference to all distance measurements. To mitigate this error and obtain more reliable distance estimates, we should depend on nodes which are very close in distance. However, choosing closeby anchors is difficult to achieve in actual environment since unknown nodes are completely unaware of their position. This can be solved by using weighting scheme on estimation.

MDS and WMDS are fundamentally multi dimensional scaling algorithms. They create visual graph of network connectivity using dissimilarity as a distance matrix. Very attractive feature of these algorithms is that they exploit distances between unknown nodes. They use anchor nodes to carefully reflect, translate or rotate the entire network graph after all computations are performed.

MMDS is the modified MDS and has better transformation matrix based on anchor node location. This technique is not fully MDS because it does not really take advantage of distances between unknown nodes. However, it performs better than MDS for a low measurement error of 0.5 dB. Increasing measurement error by 0.5 dB increases position error by 2 meters. Position error remains below 2 meters with 3 anchors and 0.5 dB measurement error.

WMDS has best performance in terms of lowest position error of below 1 meter which slightly deviates with increase in measurement error. The reason for this performance is weighted distance, exploiting distance between unknown nodes and guaranteed cost function reduction method. These three features have made WMDS most robust among all.

B. *Effect of increase in error*

Performance is measured in the network model of Figure 1 with respect to total 4 and 7 anchor nodes. Measured distance error varies up to 2 dB and peak RMSE is observed to be 8 meters in this case.

Malguki performs better with more anchors with position error much smaller than MLE and MDS. It reduces error by 2 meters at 1.8 dB with increase in anchor nodes by 3. This result indicates more anchors should be deployed in an environment where error is high to achieve comparable performance. The only critical point is to keep in mind that anchors should be placed around the periphery of the entire network. If such a placement is not followed, then Malguki gives worst performance. MLE does not take much advantage of having more anchor nodes here. It performs well with the less measurement error and with less anchor nodes. It is well suited for the scenario where anchor node quantity is less or the chances of anchor node failure is high.

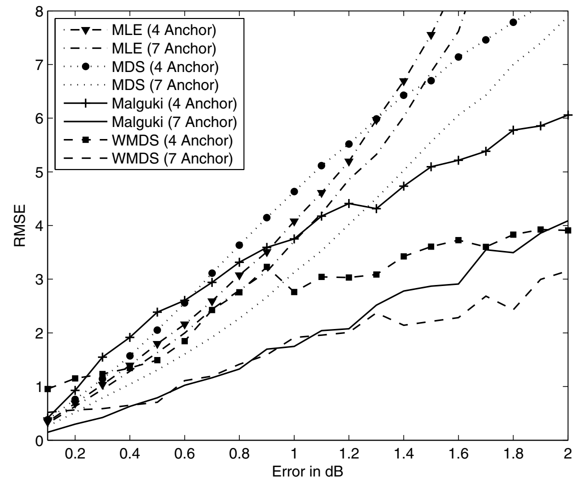

Figure 4: RMSE vs Distance Measurement Error

MMDS also has better performance with more anchors against measurement error. Position error decreases by 1 meter at 1.8 dB with 7 anchors. It is also sensitive to the large error in measurement which makes the gap between two curves very narrow at large error. Therefore, one needs to increase the number of anchor nodes with increased error to reduce position error.

WMDS has best performance among all algorithms. The measurement error has less effect on performance curve. Position error remains below 3 meters up to 2 dB measurement error with 7 anchor nodes. Increased anchor nodes have performance improvement of 1 meter at 1.8 dB measurement error. The large position error at the beginning is due to the poor initial position estimation in WMDS. However, it is clear that despite of this large initialization and measurement error, WMDS performs very well. The comparison chart in Table 1 shows performance of all algorith to make selection of an algorithm easy for a one hop network deployment.

V. Conclusion

We have implemented recently proposed localization algorithms in MATLAB and analyzed results under common network setup. Each algorithm has some limitations and some advantages. We have also demonstrated which algorithm is better under specific scenario. To achieve minimum error we need to use WMDS whereas MLE is preferred for less complexity. Both MLE and Malguki perform well when unknown nodes are

Table 1: Performance Comparison of Localization Algorithms

	MLE	Malguki	MDS	WMDS
Iteration	No	Yes	No	Yes
Computational Complexity (n is unknown nodes)	O(n)	O(nL), L is the number of iteration	O(n^2T), T is the number of steps for SVD	O(nL), L is the number of iteration
Impact of position of anchor nodes	High, Anchors should be very close in distance	High, Anchors should be on periphery of network	Less, Anchors used for final transformation	Less, Anchors used for final transformation
Max. error handling for 2-meter accuracy(with 7 anchor nodes)	0.6 dB	1.2 dB	0.7 dB	1.4 dB
Min. anchor nodes for 2-meter accuracy (with 0.5 dB error)	4	5	3	3
Limitation	Proper anchor position	Proper anchor position, convergence accuracy v/s computational time	Computational complexity due to matrix decomposition	Convergence accuracy v/s computational complexity, initial estimates of position, full dissimilarity matrix
Gives best performance When..	Unknown nodes are surrounded by anchor nodes and are very close	Unknown nodes are surrounded by anchor nodes	Anchor nodes are sufficient and moderate error in distance measurement	Initial estimation and full network connectivity are available

surrounded by anchor nodes. MDS has better performance with sufficient anchor nodes and less error. This necessitates more anchors for noisy measurements. Although WMDS gives best performance its application is limited by complexity, convergence time and initial estimate requirements. This comparison is very useful before selecting appropriate algorithm for any network.

Our future work includes verifying performance of these algorithms in actual wireless network environment. Moreover, these algorithms need to be compared using a common scenario in multi hop networks where nodes do not have direct connectivity.

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